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JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

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THE FUNCTION OF STATISTICS IN UNDERGRADUATE TRAINING[1]

By Robert E. Chaddock, *Columbia University*

The function of statistics in undergraduate training has not yet been clearly defined. As a matter of course, statistical methods have been introduced in technical and professional schools and in universities as means to accomplish desired results, as equipment for research in sociology and economics, in education, in biometry, in the practical affairs of business and social organization. Elementary courses must frequently be given in these specialized fields to post graduate students merely as an introduction to the advanced methods required for their particular work. *Statistical training has been introduced at the top of our educational scheme.* It is regarded, chiefly, as a tool. It places emphasis upon technique rather than upon a method of attack, a mode of thought, a procedure for analysis and comparison of complex factors.

The advanced student, especially in the social sciences, has left behind, usually with no regrets, the exact methods of his mathematical studies because he does not expect to apply them. *Equations,* which have aided in solving the unknown from a knowledge of the known; *logarithms,* which are effective tools in computation; *the characteristics of lines and curves,* which today are utilized effectively to describe the phenomena of social and industrial life and to illuminate their explanation—all these and much more have been forgotten because the student, as he begins his studies in the social sciences, has not been shown their significance in his work. It is only when there is little time to repair these neglected things that the professional and research worker faces the need for this technical equipment. The engineer has no such difficulty because he understands early in his training what is needed. Likewise, students in the natural sciences at the start are introduced

[1] The presidential address read at the eighty-seventh Annual Meeting of the American Statistical Association, New York City, December 28, 1925.

to the experimental laboratory. They learn to observe accurately, to assemble and classify data, to sift evidence. They begin at once to acquire the habit of independent and cautious generalization from known facts. The laboratory technique is not easy to acquire. It requires hard work. It is our belief that statistics can be made to serve for the social sciences a function similar to that which the experimental or laboratory method has served in the physical sciences.

Undergraduates who major in the social sciences have usually had at least one laboratory science, but they find that the experimental method has serious limitations in the complex relations of human society. Therefore, it is with difficulty that they carry over into their social studies scientific principles gained from their laboratory experience. The beginning work in these subjects is still too largely deductive in character. Principles and generalizations of economics and sociology are found to be too inflexible. They should be related to the concrete situations of experience and should be modified in the light of the facts. From an examination of new data new explanations arise.

My thesis is that the undergraduate majoring in the social sciences must have special training in the inductive scientific method applied to these fields, and that he should have it early in his work as a *fundamental educational equipment.* A course in elementary statistical methods should be given, as laboratory courses are now given to beginners in the physical sciences. This course would bring students into intelligent contact with the facts of our social, economic and political life, an increasing number of which are expressed in quantitative form; it would place emphasis upon accuracy of information, upon valid methods of analysis and comparison, and upon caution in generalizing; and it would aid in describing the situations of every-day life without premature theorizing.

Moreover, approaching social studies mainly from the deductive point of view, *the student frequently acquires early in his work faulty habits of using quantitative data and of thinking about them.* He is likely to call to his aid only such facts as illustrate or confirm conclusions arrived at deductively. In this case he neither verifies the truth of his premises nor makes adequate tests of his conclusions.

Inductive methods should go hand in hand with the deductive. The one cannot be separated from the other without marring the effectiveness of both. The use of the inductive method requires rigorous training in scientific procedures. We are permitting the undergraduate in the social sciences to become lost in a wilderness of concrete data and badly supported general statements without showing him how to find

his own way about. He has not yet acquired the wariness and wisdom that scientific training gives, and he is lured constantly on false trails, due to the bias of his hopes and enthusiasms. We have an obligation of first importance to remedy this situation. The elements of statistical method given as a fundamental discipline to undergraduates will furnish them with the broad foundations for scientific work and will cultivate their critical judgment of facts and explanations offered by others.

For this purpose statistics should not be conceived in any narrow or professional sense. It is not merely technique or a branch of mathematical science. It is concerned with a body of principles and methods developed to guide the student in assembling and in handling quantitative data. It is a point of view, a method of attack. It involves measurements, countings and estimates, a careful record, intelligent groupings, discriminating analysis, logical comparisons, clear presentations, and cautious weighing of evidence. There are principles and procedures common to many different fields of work. *These constitute the unity of the subject.* We can leave the special methods to be developed and applied to particular problems by professional and research workers.

Recent changes, both in the emphasis on particular subjects in the college course and in the methods and content of these subjects, have made clearer the need for a broad elementary training in statistics for undergraduates. The older course of study, during the first two years, carried forward subjects begun in preparatory and high school. The student was trained in formal logic, in deductive thinking, in the precise methods of mathematical demonstration, in the laboratory technique of the natural sciences, in the broad historical backgrounds from which current social, economic, and political movements have emerged. Upon this educational foundation was built the work of the last two years of college in the social sciences—government, economics, sociology, and the philosophy of history. Emphasis was placed not upon specialization but upon relationships in a complex society. The aim was broadly cultural. The lines of demarcation between subjects could not be sharply drawn. Perhaps even now they cannot be.

The methods of study in sociology and economics have been historical and comparative and largely deductive. Premises and assumptions were laid down, and logical reasoning was depended upon to evolve a system of theory and explanation of phenomena. The facts of experience were appealed to usually for illustrative purposes or for precise description, not for the purpose of verifying premises or

modifying conclusions in the light of new data. Formal logic, even if effective training for deductive reasoning, gave little help to the student in "checking up" his conclusions. It remains for training in statistical methods to tie up logical processes directly with the concrete facts of every-day life.

The viewpoint of college educators has changed in response to the practical spirit of the times. Early specialization is now the tendency. What was formerly post graduate or professional study is now well advanced before the student receives his bachelor's degree. In the last two years of college he selects courses which prepare him for specific professional work—business, engineering, medicine, law, teaching. The program of studies in the first two years of college has been modified to meet this new situation. The fundamental preparation, technical and cultural, formerly distributed over four years, is more closely confined to the first two years of the college course.

Instruction in the social sciences is begun in the high school. Elementary courses in economics, sociology, and government are being introduced in the Sophomore year in college or earlier. Mathematics and the classical languages are being displaced, so far as social science students are concerned, in favor of so-called practical subjects. The effort is made to give emphasis to the interdependence of social studies by introducing a general descriptive and comparative course, as illustrated by Contemporary Civilization in Columbia College, given to Freshmen five hours per week. This course furnishes a coördinated background for special courses in sociology, economics, government, history, and philosophy undertaken in the following years. It is designed to be an orientation course in the world of ideas.

This change in emphasis upon subjects in the early years of the college course is accompanied by a change in the content and the method of approach to social studies. Inductive methods are being rapidly adopted to supplement older deductive procedures in arriving at a reasonable understanding of organized society and of human behavior. This understanding of group behavior gives a common purpose to all the social sciences which deal with its different aspects. The quantitative analysis of recorded behavior in a kinetic society requires a common method—the statistical. The accumulation of quantitative data and the development of appropriate methods for handling them have been slow. The student of even thirty-five years ago gave little attention to statistical studies because both data and methods were lacking. Recent years have witnessed a rapid development of both.

The very rapid accumulation of quantitative data descriptive of

events threatens to confuse the beginner. He is tempted to read too much and to think too little. We have speeded up his approach to social studies at the same time that the materials with which he works have multiplied in volume and complexity. There are many pitfalls in using concrete data gathered and presented by many writers and investigators for various purposes, describing conditions where the factors are many, the variations wide, and the units difficult to define. These pitfalls can be avoided only by developing a healthy skepticism of so-called facts and a knowledge of scientific methods.

Furthermore, facility in the use of scientific methods cannot be acquired by talking about them. They are acquired by doing—by trial and error and by hard work with actual data. One of the chief criticisms of the writings and discussions in the social sciences is too much dependence upon unchecked generalities and too little patient effort with the underlying facts upon which the validity of conclusions depends. Jumping at positive conclusions from insufficient or crude data is a bad habit, we all admit, but the problem is how to prevent our students from acquiring this habit. One answer is training in statistical methods as early as possible in the undergraduate preparation. Let us build our social science structure upon due regard for facts and caution in their use, and upon correct habits of thought.

Merz, in his *History of Thought in the Nineteenth Century*, after describing how the successful investigator in physical science has always been the man who could single out some special thing for minute investigation, who could retire with one definite object into his study or laboratory and there fathom and unravel its intricacies, explains the difficulties of the "practical man." "That which everywhere oppresses the practical man is the great number of things and events which pass ceaselessly before him, and the flow of which he cannot arrest. What he requires is the grasp of large numbers. He is always and everywhere met by the crowd of facts; by the relentlessly hurrying stream of events. . . . Thus has arisen the science of large numbers or statistics and the many methods of which it is possessed." Here is clearly stated the contrast between the experimental method of natural science and the statistical method of the social sciences.

The work of experimenting, when possible at all, requires a different technique in the social sciences. The investigator as a rule must rely far more on statistical methods and safeguards. He must observe a complex situation as he finds it, for the most part; he must analyze it and secure measurements if he can, not such as he would prefer under ideal controlled conditions, but such as are available; he must scrutinize

his facts and determine the representative character of the samples before generalizations can be established; and he must state his conclusions in terms of probabilities.

Statistical results describe concrete situations and relationships in a limited field. *Crude* quantitative data, widely employed at present for precise description, are likely to convey wrong impressions and to lead to errors of interpretation. Therefore, statistical devices have been invented to refine the raw data and to make our description of phenomena simpler and more reliable. Tables, graphic devices, rates, percentages, averages, index numbers, measures of dispersion and measures of relationships—the elementary methods used in statistical analysis, are devices to describe as simply and adequately as possible "the relentlessly hurrying stream of events." These simple scientific methods the undergraduate should know. How else can he hope to detect errors of fact and fallacious inferences and presentations which he encounters in the course of his daily work? Of course the ultimate aim of scientific investigation is to be able to draw inferences from established facts and relations, to arrive at a sound scientific argument and to detect a false one. The statistical procedures, however, are preliminary to this final step—the results of statistical analysis are the foundation upon which the inferences rest. All too often conclusions are formulated before the facts are examined.

It is not the purpose of this paper to set forth in detail the *content* of a course in elementary statistical methods for undergraduates. We wish only to inquire what *functions* such a course should serve in college training and why it should be given in college at all. If all the social sciences are beginning to use statistical methods, clearly, this beginning course should not be specialized in any single field. It should emphasize methods and procedures which are applicable in any or all fields. Why should we duplicate teaching effort by giving the same elementary principles of statistics in each of the social sciences? This course offers an opportunity to emphasize the relationships between the subjects taught and to develop unity in the social sciences by the use of a common method. Only incidentally should the course convey factual information. The facts chosen from various sources familiar to the student's experience should serve *primarily* as the basis for training in scientific methods. The object is not to give to the student simply a command of formulas and processes. The use of the statistical method, as of scientific methods generally, requires constructive imagination and ability to guard against wrong inferences and ability to detect them in the work of others. The usual sources of errors are of a very elementary sort.

We shall now summarize some of the most important services which elementary statistical training can render to the undergraduate:

1. *Develops the critical judgment of the student.* Figures may speak for themselves but not necessarily the truth. The student should learn their language. He requires principles, not difficult but simple ones, which enable him to judge of the credibility of quantitative results. The independent, thinking student must cultivate skepticism of his own conclusions and those of other people. He faces masses of quantitative data in so-called statistical reports. He should be able to discriminate between *reliable* and *unreliable* sources. By studying examples of misuses of statistical data he can develop the ability to discover fallacies and sources of errors.
2. *Emphasizes the essentials of scientific method.* Science is getting at facts and trying to understand them. The scientific attitude of mind prompts the student to seek evidence which will appeal as true to other minds as to his own. He must ask, What do the figures represent? How were they collected? What is included in the table or chart? Are the data relevant to the situation described? Are they adequate? What are the shortcomings of the data? Are they comparable? Are they properly classified? Classification is the foundation of scientific knowledge. Especially in the social sciences where factors are many and varied and where human sympathies are keen it is difficult to free judgments from the bias of desires and feelings —"*wishful thinking*" Professor Giddings calls it. Quantitative data are objective and can be freed more easily from bias than subjective opinions.

We would not make professional statisticians out of Sophomores or Juniors in college if we could, but elementary training in statistics offers the opportunity to work with concrete data and to develop the inductive method of attacking problems.

3. *Prepares for advanced work.* Early statistical training prevents the development of bad habits in the use of quantitative data. It provides the student with the fundamentals of the necessary technique for professional work in both the natural and the social sciences. It develops a scientific attitude of mind as a part of his educational equipment. If he does not contemplate advanced academic work it gives him an intellectual equipment highly valued in the world of business and social organization.

4. *Helps the student to realize the interdependence of different factors in a given situation, and to describe relationships of cause and effect.* The factors of a complex situation can be observed, measured, classified and compared, even when they cannot be controlled for a genuine experiment, as in the natural sciences. Descriptions and comparisons of complex situations require the use of statistical methods.
5. *Supplements deductive processes.* Furnishes means of "checking up" general statements. Ties up logical processes with concrete situations, and emphasizes logical consistency. The quality of mind we call "common sense" is really logical consistency. We are not born with it: it must be cultivated. Statistical training aids in its development.
6. By making and interpreting graphic representations of statistical data and by constructing tables, the student gains an understanding of the *continuity* of series of variables and their relations to each other. He begins to observe the *stability* in results obtained from the analysis of large numbers of cases. He realizes that the basis of belief in future occurrences lies in past experience. This is the fundamental concept of prediction. It is probability, not certainty.

   The making and using of graphs also develop the critical abilities of the student. The fallacies are many and often concealed in the data themselves, as well as in the methods of constructing the diagrams. Visualization of the data aids in discovering errors and in suggesting new ideas.
7. Finally, an elementary course can preserve useful mathematical concepts by showing their application to social data.

THE STATISTICAL ALLOCATION OF JOINT COSTS [1]

By W. L. Crum, *Harvard University*

There are numerous industries in which the circumstances of production are such that all or a considerable portion of the costs of operation cannot, apparently, be apportioned among the several products and must therefore be charged to those products jointly. Such costs are generally called *joint costs*, and are of different sorts according as the joint production of the several products is in fixed proportions or variable proportions.

Broadly speaking, instances of fixed proportions are found in the extractive industries, whereas the proportions are ordinarily subject to discretionary control in manufacturing. In mining especially, the ratios between the amounts of the several minerals in the given body of rock are not subject to control, although a large operator might conceivably work only those veins in which the ratios were favorable. In agriculture also the fixity of proportions between the several products is generally rigid, but here some variation may be achieved slowly by processes of breeding or of adjusting the environment. In manufacturing, on the other hand, control of the relative volumes of the several products is more considerable, although even in the fabrication of goods from raw materials there are exceptional instances of true by-product conditions in which the production of a given volume of one commodity implies the production of a given volume of another.

Under either type of joint production it is now held that an individual operator cannot allocate some or all of the costs of operation to the several commodities. Some at least of the cost of production is held to be joint, and not directly chargeable to either (or any) of the commodities. In such cases the attempt to compare prices with cost of production is frustrated by the inability to ascertain the fraction of the joint cost which should be assigned to the particular commodity.

In many cases some of the total cost of production is directly assignable to the several commodities, and only the remainder is joint cost. For true by-product industries, where the production proportions are rigidly fixed, all the essential costs are joint. Here many costs which appear to be directly due to the one commodity are essentially joint,

[1] A paper read at the eighty-seventh Annual Meeting of the American Statistical Association, New York City, December 30, 1925.

because the one commodity must be produced in a definite ratio by volume with the other commodities; but there are direct costs, as for example the normal cost of packing and marketing. Any cost apparently involved in producing the one commodity could not be eliminated, or measurably reduced, without curtailing the production of the commodity group as a whole.

Where the production proportions are subject to control, however, the proportionate volume of production of one of the commodities will tend to be reduced until the price realized covers at least the costs directly assignable to the production of that commodity. Under these conditions profitable production can continue if the price of each commodity is sufficient to cover at least the direct costs involved in its production, and the prices and volumes of the several commodities are such that the excess of return over the direct costs amounts for the entire group of commodities to more than the total joint cost.

Actual cases of the absolute by-product condition, with rigidly fixed production proportions, are extremely rare; and most practical cases of joint production involve direct costs as well as joint costs. Some direct costs are obvious, and can be assigned by the familiar methods of cost accounting.[1] It is customary to include all other expenses as joint costs; and yet, if more effective means than a system of cost accounting were available to isolate them, it is quite likely that some or perhaps all of these expenses might be assignable as direct costs of the several products.

Conceivably an individual operator might, by a process of experimentation with different production proportions, learn how his aggregate costs varied with the varying proportions and devise from the resulting data an empirical rule for allocation of part or all of the joint costs. Such a process of experimentation, however, would be so lengthy that the individual could scarcely afford it and that the results would probably be vitiated by those changes, slow indeed but important in any considerable time interval, which are continually going on. What is needed is an approximately instantaneous set of observations of the aggregate costs for production with different production proportions. Such observations are obtainable if there is a fairly large group of operators each using his own production proportion, and all keeping detailed and uniform accounts. It is probable that conditions other than the production proportions, which affect the total cost, will differ for the different operators. If however there is no systematic relation between these differences and the differences in

[1] The methods of cost accounting referred to do not include a system which apportions joint costs by rule-of-thumb, even if such apportionment be determined from long observation.

production proportions—if these other differences are random and accidental in their distribution, so far as the immediate problem is concerned—any persistent relation found between the aggregate cost and the production proportions can be accepted as of some significance for the assignment of costs.

An excellent example of joint production meeting these requirements is found in the American railway transportation industry. A detailed and thoroughly uniform accounting system affords data on the volume of freight traffic, the volume of passenger traffic, and the aggregate cost of transporting these volumes, for a large number of operators. Mention will be made, in Section V, of certain other industries in which the analysis would be possible if the essential uniform accounts were available, and of certain further applications of the analysis to the railway industry, which could be made if adequately detailed records were kept.[1]

I. RAILWAY FREIGHT AND PASSENGER TRAFFIC

In the conduct of railway transportation there are some costs which are obviously assignable to passenger traffic, and some to freight traffic. The other costs, somewhat less than half of the total according to various estimates, appear to be joint. It is held that, if railway management were free to make rates in its own interests, the average rate for each of the two sorts of traffic would bring a return on such traffic equal at least to the direct cost involved and contributing something if possible to defray the joint costs. Moreover, the aggregate return from both sorts of traffic (ignoring other sources of income, which are relatively unimportant) would need to be sufficiently in excess of all direct costs to cover all joint costs.

It is of importance in the rate making problem to know whether any part of the so-called joint cost is separable into direct costs and thus assignable to the two classes of traffic individually. Clearly the smaller the portion of the aggregate cost which is truly joint the less chance exists for the burdening of one class of traffic with an unfair share of the total expense of operation. Presumably it is desirable to charge each class of traffic a rate which will pay the full cost of moving that traffic; and, although reasonable doubt may exist as to the wisdom of basing action on this principle and it may be extremely difficult to ascertain the facts, there is an unmistakable tendency to examine rates in the light of the cost of service—direct cost plus an

[1] A considerably more detailed statement of the joint cost problem will be found in Chapter 7 of Ely's *Railway Rates and Cost of Service* and in the articles by Professor Taussig referred to therein. Some comment upon the problem will be found also in Chapter 7 of Vanderblue and Burgess's *Railroads: Rates, Service, and Management.*

appropriate fraction of joint cost. The Interstate Commerce Commission, for example, has from time to time made some study of the problem of allocation and has suggested certain rule-of-thumb schemes of apportioning the joint expenses.[1]

The analysis is confined to Class I railways; and, with few exceptions, the specific examinations cover all such reporting railways in the United States.[2] Annual data are used, the figures covering reporting years (July to June) before the War and calendar years after the War; and the official publications of the Interstate Commerce Commission are taken as the source of statistics. The statistical methods used are those of forming frequency series, measuring the means and dispersions of such series, and examining ordinary and multiple correlation and regression relations between variable magnitudes. Although outline comment on the statistical methods accompany the textual discussion, comprehensive exposition of these methods should be sought in some of the numerous texts on statistical method.

II. TON-MILE AND PASSENGER-MILE COSTS

The principal examination comprises a study of the data for each of several years with a view to establishing an equation stating the relation between expenses per mile on the one hand and ton-miles per mile and passenger-miles per mile on the other.[3] The essential data are found in the annual volume on *Statistics of Railways in the United States* published by the Interstate Commerce Commission, and in particular they appear in Section A, Part I, entitled "Abstract of Reports Rendered by Operating Steam Railway Companies of Class I."[4]

The first step in the analysis for any one year consists in forming three frequency series (number of miles being taken as frequency), one each for expenses per mile, ton-miles per mile, and passenger-miles per mile. As might be expected the variable in each such series shows a wide range of variation for the railroads as a group, and further evidence of the extent of this dispersion among the operating com-

[1] Interstate Commerce Commission Reports, Volume 30, page 676, a decision was rendered concerning the separation of operating expenses, and it was held "that sufficient reason exists for requiring the extension of this separation beyond what has been required heretofore in the Commission's annual report." See page 45, 20th Annual Report I. C. C. An order was entered June 15, 1915 by the Commission prescribing rules for the separation of expenses. This order was modified because of the War, October 23, 1917; but, effective January 21, 1920, the rules with amendments were restored. See page 36 of the 34th Annual Report I. C. C.

[2] Class I railroads include those with gross earnings annually of $1,000,000 or more.

[3] In the analyses of Sections II and III, Miss A. E. Ohse, of the staff of the Harvard University Committee on Economic Research, rendered valuable assistance.

[4] In the 1923 volume, for example, the items for each railroad appear in "Statistics of Rail Line Operations" and are numbered 1 (average mileage of road operated), 89 (operating expenses), 91 (ton-miles-revenue freight), 93 (passenger-miles-revenue). Similar items can readily be located in the volumes for the other years.

panies appears in the items *coefficient of variation* in Table I.[1] Clearly, then, the railway system affords an instance of non-uniformity in volumes of production and in expenses. Moreover, as is well known, the relative densities of freight and passenger traffic differ considerably for the various carriers; and statistical evidence of this fact can be found quickly by calculating the ratio of passenger-miles per mile to ton-miles per mile for several companies. Thus there is sufficient diversity in the expenses and the traffic proportion for the different railroads to render the observed facts for the various railroads equivalent to the results of an experimentation with different traffic proportions. The next question is: Does a systematic relation exist between the expenses and the traffic proportion?

TABLE I

AVERAGE AND COEFFICIENT OF VARIATION, FOR EACH YEAR EXAMINED, FOR EXPENSES (*E*), FREIGHT TON-MILES (*F*), AND PASSENGER-MILES (*P*)

	Average for All Roads*			Coefficient of Variation		
	E	*F*	*P*	*E*	*F*	*P*
1911–12	840	115.0	14.73	64	94	91
1913–14	920	116.8	15.87	67	91	88
1919	1865	155.8	20.00	70	81	101
1921	1936	131.7	16.11	68	81	121
1922	1858	142.7	15.10	70	79	131
1923	2022	174.3	15.42	73	85	127

* Units: $1 for *E*, 1000 ton-miles for *F*, 1000 passenger-miles for *P*.

An obvious method of attacking this question consists in seeking, by the method of correlation, a systematic relation between the expenses and the ratio of passenger traffic to freight traffic. This test is not made because it seems probable that the relation might be obscured or distorted by the great variations in the actual—as against the relative—volumes of traffic. The procedure adopted therefore is to find the multiple regression equation between expenses per mile as the dependent variable and ton-miles per mile and passenger-miles per mile as the independent variables.[2]

[1] The coefficient of variation for a given statistical series is the ratio of the standard deviation (which is the standard measure of dispersion) to the arithmetic mean, and is usually expressed in the form of a percentage. The object of using the coefficient of variation, rather than the standard deviation proper, is to have an indicator of dispersion which is entirely free from the units of measurement used.

[2] Such an equation expresses the dependent variable as equal to the sum of three terms as follows: a factor multiplied by the first independent variable, a factor multiplied by the second independent variable, and a further term which appears as a fixed constant. Although it is possible for the regression equation to have a more complicated form than this, the simple relation here outlined is fairly typical. The use of the word independent in connection with any of the variables is somewhat misleading: all the variables are connected with each other in a more or less rigid manner, so that none is independent except in the mathematical sense.

In the analysis linear regression is assumed; for, although there is some evidence that the relations are not truly linear, it does not appear that the available data justify the use of more complicated regression curves. Outline correlation tables between pairs of the variables (per mile figures throughout)—expenses with ton-miles, expenses with passenger-miles, ton-miles with passenger-miles—can be formed for each of the years.[1] These tables suggest a highly linear relation between expenses and freight traffic, but somewhat less regular relations between expenses and passenger traffic and between freight traffic and passenger traffic. Further comment on this condition appears in Section IV.

TABLE II

ORDINARY COEFFICIENT OF CORRELATION AND REGRESSION, FOR EACH YEAR EXAMINED*

	Coefficient of Correlation			Standard Deviation†			Coefficient of Regression		
	E with *F*	*E* with *P*	*F* with *P*	*E*	*F*	*P*	*E* on *F*	*E* on *P*	*F* on *P*
1911–12	.876	.768	.479	539	108.6	13.42	4.34	30.8	3.88
1913–14	.856	.794	.511	616	106.4	14.02	4.96	34.9	3.88
1919	.881	.773	.490	1308	126.5	20.26	9.12	49.8	3.06
1921	.829	.733	.383	1322	106.4	19.54	10.30	40.0	2.09
1922	.848	.698	.357	1297	112.3	19.70	9.78	45.9	2.04
1923	.862	.636	.323	1480	148.0	19.74	8.62	47.8	2.43

* *E* refers to expenses; *F*, to ton-miles; *P*, to passenger-miles.
† Units: $1 for *E*, 1000 ton-miles for *F*, 1000 passenger-miles for *P*.

The first operation is to compute the ordinary correlation coefficients, which appear in Columns 1–3, Table II; and from these the regression coefficients are found, Columns 7–9.[2] Knowing the regression coefficient and the mean values of the variables, it is possible to set up the equation of estimate (ordinary regression equation) of which a typical instance is (expenses per mile estimated from freight traffic density, 1911–12):

$$E = 4.34\,F + 341$$

An equation of this sort gives the average relationship between two of the variables, the effect of the third being concealed. As it is desired to reveal the individual effects of the two independent variables, freight traffic and passenger traffic, on the dependent variable, ex-

[1] In the specific correlation tables (or rather scatter diagrams) which were used, each individual road was represented and no distinction was made between roads according to mileage, except that roads which were unusually short were indicated in red and roads which were unusually long were indicated in green, intermediate lines being indicated in black. It was thus possible to make some allowance in the visual appraisal of the scatter diagram for the differences in length of road.

[2] Regression coefficient of expenses (E) on freight traffic density (F) is given by $r\,\sigma_E/\sigma_F$ where r is the correlation coefficient, and σ is the standard deviation.

penses, the multiple regression equation is required. From the correlation coefficients and standard deviations in Table II, the partial regression coefficients are directly calculable, and the multiple regression equations of Table III are established by the use of these coefficients.[1]

TABLE III

THE MULTIPLE REGRESSION EQUATION, FOR EACH YEAR EXAMINED, EXPENSES BEING REGARDED DEPENDENT*

1911–12	$E = .0033F + .018P + 193.3$
1913–14	$E = .0035F + .021P + 171.7$
1919	$E = .0068F + .029P + 221.3$
1921	$E = .0080F + .033P + 353.0$
1922	$E = .0079F + .030P + 279.2$
1923	$E = .0073F + .030P + 285.4$

* Unit: $1 for expenses, one ton-mile for freight, one passenger-mile for passengers.

One of these equations, for example that for 1911–12, affords an estimate of the expenses per mile based upon a knowledge of the ton-miles per mile and passenger-miles per mile (the possible effects of other variables being concealed). In consequence of the manner of determination of the equation, such estimate is a fair approximation to the actual figure; and, although for a particular road the deviation of the estimated from the actual expense may prove considerable (see Table IV), the deviations for a number of roads tend to offset one

TABLE IV

COMPARISON OF ESTIMATED WITH ACTUAL EXPENSES PER MILE, FOR SELECTED ROADS IN 1923

	Freight Ton-miles	Passenger-Miles	Expenses	
			Estimated	Actual
B. & O.	3,959,439	177,678	$34,500	$37,583
W. Md.	3,008,470	67,081	24,300	22,023
Monongahela	4,831,039	117,270	39,100	37,122
Southern	1,165,298	140,307	13,000	16,126
Virginian	5,515,397	52,124	42,100	37,571
A. T. & S. F.	1,104,110	160,378	13,200	16,265
Ore. S. L.	1,092,372	70,428	10,400	12,022

another. In other words, the equation affords a satisfactory estimate *on the average*, and is therefore a satisfactory statement of the dependence of expenses per mile on freight and passenger traffic density *on the average*.

[1] The multiple regression coefficients, for variables measured from their means in units of one standard deviation, are found by the determinant method outlined in Kelley's *Statistical Method*, Chapter XI, and then adjusted for the "natural" units.

The immediate significance of the equation for 1911–12 is that an increase of one ton-mile in freight traffic density increases the expenses per mile by .33 cents, and an increase of one passenger-mile in passenger traffic density increases the expenses by 1.8 cents. It should be emphasized that this is an *average* condition: for a particular road it may be quite untrue, but on the average it is approximately accurate. Moreover, it should be noted that the conclusion concerns *increases* (or decreases) from a situation not greatly different from that existing at the time of analysis. Although a definite equation between the three variables is presented, that equation is not necessarily valid except for moderate fluctuations of the variables from their values as given by the statistics actually examined.

Somewhat more detailed comment on this last point is desirable. The essential fact is that an equation derived from the simultaneous observation of a large number of cases is being used to infer a relation between the variables during successive stages (in time) for the average case. Such inference is presumably valid so long as these variations from one stage to a later stage are not sufficiently important to render erroneous those coefficients of regression upon which the equation rests. This implies that the departure from the general configuration, of expenses and traffic, originally examined cannot be more than moderate. In other words, the above conclusion applies on a margin: given such a situation as actually existed in 1911–12, the expense incident to a moderate increase in traffic of either sort is approximately as indicated by the equation.

Suppose the traffic conditions in successive stages did differ markedly from those of preceding stages, because of changes in rates or other influences. The equation stating expense as dependent upon the densities of the two types of traffic would then change: a minor change of this sort appeared between 1911–12 and 1913–14, and a very marked change occurred between 1913–14 and 1919, but from 1921 on considerable stability is evident (Table III). This does not mean that from 1912 to 1921 no satisfactory allocation of expense was possible, it does mean that in those years conditions in the transportation industry were in a state of flux rendering any particular rule of allocation obsolete a short time after the year upon which it was determined.

Objection might be raised that establishment of rates in accordance with an allocation equation such as that for 1923, for example, would automatically induce such a change in the traffic densities as to render the allocation equation no longer valid. The answer is that a new equation would be found and new rates established, and that the proc-

ess of adjustment—analagous to the ordinary adjustment of production and consumption, of supply and demand—would proceed until ultimately a state of equilibrium would be reached. At some intermediate stage of the adjustment process a new factor might enter the situation, an invention or a wage change or a migration of industry, and inaugurate a new process of adjustment on the altered basis. Such an experience however is not peculiar to this problem: it is a common and familiar fact in economic problems. Its presence here merely deprives the solution for the allocation problem of that *in vacuo* character which one might hope to have it enjoy.

TABLE V

COMPARISON OF RATES AND ALLOCATED COSTS, PER TON-MILE AND PER PASSENGER-MILE, FOR EACH YEAR EXAMINED *

	Freight Ton-Mile		Passenger-Mile	
	Rate	Allocated Cost	Rate	Allocated Cost
1911–12	.0073	.0033	.020	.018
1913–14	.0072	.0035	.020	.021
1919	.0097	.0068	.025	.029
1921	.0128	.0080	.031	.033
1922	.0118	.0079	.030	.030
1923	.0112	.0073	.030	.030

* Unit: one dollar.

Table V compares the average rates received for freight and passenger traffic with the corresponding allocated costs. It is immediately evident that, in all the years studied, passenger traffic just about paid its way, the discrepancies being mainly on the side of a deficiency in rates. It must be remembered that these figures portray average conditions; and, as all available evidence points to a very considerable diversity of conditions for the various roads, the conclusion seems warranted that a very large part of the passenger transportation in those years was provided at less than the actual cost of service.

For freight traffic, on the other hand, the average rate was always above the allocated cost; and it was the freight service, therefore, which provided, on the average, a very large part of the net income of the transportation industry. As in the case of passenger service, these figures show average conditions; and there was undoubtedly great variation among the different roads. Hence it is quite probable that some roads actually conducted freight service at a loss. Moreover, there is another type of variation concealed by this analysis, the differences in rate—and in cost of service—for different commodities. Further remarks on this point will appear in Section IV, but it should

be remarked here that the figures of Table V by no means preclude the probability that certain classes of commodities were transported at a loss by some or even by many railroads.

In examining Table V, we note that the maximum defect of passenger rates occurred in the year 1919. Likewise in that year the relative excess of freight rates above costs allocated to freight service was at a minimum. The record as a whole, for the years studied, indicates a progressive fall of rates as compared with costs until 1919, followed by a gradual and (except for freight in 1922) progressive rise of rates as compared with costs. Although the relations which existed in 1911–12 had by no means reappeared in 1923, there has undoubtedly been some progress toward a restoration of those relations between rates and costs which are favorable to the profitable conduct of the transportation industry as a whole.

It may fairly be contended that the determination of rates in such manner that the average freight and average passenger rates would be about equally above the corresponding allocated cost of service would result in such a modification of the relative traffic proportions as completely to alter the allocation equation and call for a new determination of rates. There is nothing impracticable about this: the process of adjustment mentioned above could go on until an equilibrium had been reached. During the process, however, the railroads might suffer a serious impairment in total income, without a completely offsetting reduction in total expense, because of the driving away of some traffic which would not stand the higher rates. On the other hand, it may fairly be held that there is at least as much chance that the corresponding reduction of certain rates would call forth such an increase of business as actually to improve the lot of the railroads. The question can scarcely be answered in advance—as in other price problems, the method of trial and error alone would afford definitive answer. In any case, this point bears not upon the problem of allocation of joint costs but upon the wisdom of fixing rates in the light of such allocation. That there is a reasonable doubt whether such rate fixing is economically wise was remarked in the introductory paragraphs of Section I.

III. SUPPLEMENTARY ANALYSES OF RAILROAD DATA

It appears that a considerable portion of the diversity of conditions, as among railroads, may be due either to systematic regional peculiarities affecting the transportation industry or to the inclusion in the group studied of some railroads with exceptional traffic proportions.

To examine the effect of these influences, two special computations are presented for 1911–12: one for all Class I roads in the Eastern District, and one for all Class I roads excepting fifteen unusual cases.[1]

The principal results, compared with corresponding figures for the original study of all 1911–12 data, appear in Table VI. Evidently, for the eastern roads the cost of freight service was somewhat greater and

TABLE VI

COMPARISON OF RESULTS OF THE GENERAL ANALYSIS COVERING 1911–12 WITH CORRESPONDING DATA FOR SELECTED ROADS

	All Roads	Eastern Roads	All Except 15 Roads
Mean values:*			
Expenses	840	1464	819
Freight	115.0	233.2	113.2
Passengers	14.73	27.39	13.80
Coefficients of variation:			
Expenses	64	49	63
Freight	94	62	92
Passengers	91	77	71
Correlation coefficients:			
E with F	.876	.822	.921
E with P	.768	.651	.742
F with P	.479	.239	.650
Partial regression coefficients:†			
E on F	.0033	.0035	.0037
E on P	.018	.016	.013

* Unit: \$1 for E, 1000 ton-miles for F, 1000 passenger-miles for P.
† Unit: \$1 for E, 1 ton-mile for F, 1 passenger-mile for P.

the cost of passenger service was somewhat less than for the entire United States. When, however, certain exceptional carriers are left out of consideration in the allocation of costs for the country as a whole, the cost of freight service is even greater and the cost of passenger service less than for the eastern roads. In neither case is the difference large, and a study of the data of Section II suggests that a moderate lapse of time may develop differences quite more substantial than those revealed by Table VI.

An additional study seeks, for each of the years 1911–12 and 1913–14, to ascertain the relation between expenses per mile on the one hand, and freight revenue per mile and passenger revenue per mile on the other hand. Data for this examination are taken from the annual summaries in the monthly publication of the Interstate Commerce Commission entitled *Bulletin of Revenues and Expenses of Steam Roads*

[1] The fifteen railroads excluded are: Long Island; Monongahela; Philadelphia, Baltimore, and Washington; West Jersey and Seashore; Pittsburgh and Lake Erie; New York, New Haven and Hartford; Atlantic City; Lehigh and Hudson River; Port Reading; Elgin, Joliet and Eastern; Bessemer and Lake Erie; Pittsburgh, Shawmut and Northern; Virginian; Duluth, Messabi and Northern; Duluth and Iron Range.

in the United States.[1] The analysis proceeds along lines similar to those followed in studying the figures relating to volume of traffic; and the results, compared with figures previously obtained for the same years from the other data, appear in Table VII.[2]

The findings are quite striking. In each year the allocated freight cost on this basis exceeds that found on the basis of the traffic density

TABLE VII

ALLOCATED COSTS, IN 1911–12 AND 1913–14, BASED UPON AN ANALYSIS OF TRAFFIC DENSITY AND AN ANALYSIS OF REVENUES

	1911–12		1913–14	
	From Density Data	From Revenue Data	From Density Data	From Revenue Data
Dollars per ton-mile	.0033	.0050	.0035	.0053
Dollars per passenger-mile	.018	.013	.021	.019

data. On the other hand, in each year the new allocated passenger cost is less than the old. Why should the two analyses give unmistakably different results?

The chief reason for the divergence is probably to be found in the lack of uniformity between roads as to specific rates, length of haul, and constitution of traffic. On the other hand, it is quite likely that neither determination of the allocation equation is highly precise, because of statistical irregularities which are present in such number and intensity as to limit the effectiveness of the correlation analysis. Further comment on this phase of the comparison appears in Section IV. It is entirely possible that a similar experiment for some other year might show more nearly equal allocation factors by the two methods, and there is no evidence indicating that the method based on revenues could not give a lower freight factor and a higher passenger factor than the method based on physical volumes of traffic. Certainly the differences in result, as between the two methods, are sufficient to suggest the impossibility of using one method as a means of estimating results obtainable by the other.

[1] For example, in the issue for June 1912 the items taken were freight revenue per mile, passenger revenue per mile, total operating expenses per mile, average miles operated for each railroad. On page 9, for instance, the data used are given in the second block verically under Pennsylvania Railroad Company in the group of columns.

Assistance in the compilation of these figures was rendered by the author's father, Mr. S. H. Crum.

[2] In compiling the allocated cost per mile for freight traffic according to this method, account was taken of the published average rate per ton-mile. In other words, the equation arrived at by the analysis was an equation between dollars of expenses and dollars of revenue, and the dollars of revenue item was converted to physical units by use of the published average rate. A similar operation was used for passenger traffic. The fact that average rates were not actually realized by individual railroads introduces an element of discrepancy.

There is a further application, than the comparison of rates with allocated costs, which can be made of the method outlined above. The regression equation for any year, say 1911–12, may be taken as setting a standard for expenses: given a certain density of freight traffic and a certain density of passenger traffic, the equation indicates a definite level of expenses per mile. As suggested earlier, the regression equation applies to average conditions for the country as a whole, and it by no means follows that the expenses and traffic of a given road must satisfy this relation (Table IV). On the other hand, as failure to satisfy the average relation must be due to conditions peculiar to the particular road, the fact of such failure should call for an explanation. If, for a given road, the expenses and traffic densities do not satisfy the equation, an explanation should be sought in special conditions which set this road apart from the average road. If such special conditions are not found to exist, the difference of actual expense from estimated expense may be due to unusual efficiency or lack of efficiency in operation. In this sense, the equation is an operating standard, and, with proper allowance for its limitations, may prove of great usefulness in the conduct of transportation.

IV. MISCELLANEOUS COMMENT ON METHOD

There are numerous technical points of interest touching the applications of correlation, and other statistical methods made in the preceding sections. An important obstacle to the most satisfactory statistical analysis which might be desired lies in the fact that the data apply to roads as a whole. For example a great system, like the New York Central or the Southern Pacific, may easily be affected by as much diversity in the expenses or traffic density of its numerous operating units as is revealed among the many small roads for which individual data are obtainable. For each reporting road then, whether it is a short compact line or a great ramifying system, a single average figure is used for each of the variables under examination. This fact undoubtedly detracts from the effectiveness and adequacy of the analysis.

There can be little question that, if data were available for the various homogeneous divisions of the great systems as well as for all the separately operated roads, a more conclusive attack upon the problem could be made. Moreover, the results would presumably be quite different from those found from the data actually available. So great is the expense and inconvenience incident to compiling all the statistics now required in the transportation industry that one hesitates to suggest that further or more detailed figures be reported.

Nevertheless, it is believed that the useful results obtainable from a more complete analysis of the sort outlined above, resting upon more detailed data pertaining to the minor operating divisions regardless of ownership or control, would be sufficiently useful to management as well as to the public to justify the trouble entailed.

That the use of the average figures pertaining to individual roads as a whole affects the statistical results obtained can scarcely be doubted. The dispersion existing within any great system would, if it were taken into account, probably alter the computed dispersion for the entire group of railroads. That such alteration would be large however is doubtful, for there is a good chance that the variation within systems has such form and intensity as to minimize the effect of its being concealed in the coefficients as calculated. Likewise for the measures of correlation. Although for these statistical numbers there is much more chance that the actual consideration of data relating to separate divisions would appreciably modify the results, there is little reason to believe that the main conclusions would be modified.

The question of the form of the regression relationships among the variables is quite perplexing. Examination of scatter diagrams shows that, although the main tendency is distinctly linear, some indications of curvature and much general irregularity appear. The fact that few of the regression equations pass through or very near the origin—that when traffic is approximately zero estimated expense is not approximately zero—further suggests that the appropriate relation should be curvilinear. In view, however, of the substantial obstacles in the way of obtaining highly precise results from the available data, it appears that the assumption of linear regression is a sufficient first approximation. On this assumption much that is worth while and much that is suggestive of further analysis can be worked out. Afterwards, with more detailed data or more intensive methods of analysis, the allocation equations can be refined and perfected.

Although the study as presented pertains to allocation of expenses between passenger and freight traffic, similar methods can be used to apportion costs between different types of freight traffic. With appropriate data considerable progress can be made in determining what fractions of expenses are assignable respectively to bulky raw materials as against manufactured commodities, to long haul as against short haul traffic, to traffic received from connections as against traffic loaded and delivered within the division. Moreover, the specific expenses, such as maintenance of way and structures or maintenance of equipment, can be allocated in a more or less satisfactory manner by the same methods. In every case of this sort, the allocation

equation affords also an operating standard, indicating the average or "ideal" level for expenses according to given traffic volumes. Too much must not be expected in these directions, but results of sufficient importance for management amply to justify all the cost of analysis should easily be obtained.

In the use of revenue data, instead of traffic density, as a basis for the analysis many hidden factors are introduced which are not present when physical figures alone are used. The differences in rates and traffic structure are perhaps the most important in affecting the relation of revenue figures to density figures, and an important indirect effect also enters through the relation of length of haul and size of load to expenses. Although it is presumably possible to secure and analyze data which would enable the investigator to account for these disturbing factors—and discover much of interest besides—it is unlikely that at best the revenue data would yield results closely confirming those resting on physical data. The real difficulty is that the diversities are so great and the causes of diversity so numerous that no statistical processes are sufficiently powerful to isolate exact relations holding between expenses and a few only of the basic causes of fluctuation.

This view bears upon any use of correlation processes in the allocation of expenses—the use of these methods presupposes that other causes of fluctuation than those under examination tend to cancel each other when the number of roads (or divisions) studied is great. That such cancellation occurs in large degree can scarcely be doubted, and it is upon this hypothesis that such conclusions as are drawn from the allocation study must rest. That such cancellation is sufficiently complete to warrant very detailed conclusions is doubtful, and it becomes more doubtful as the object of analysis changes from the general to the specific.

V. CONCLUSION

By use of available data on expenses and traffic volumes it is possible to develop certain statistical summaries which yield equations for the allocation of costs between freight and passenger traffic. This allocation is not highly precise, partly because relationships are not truly represented by straight lines and partly because the data pertain to entire railroads rather than to individual operating units. Nevertheless, precision is probably sufficient to warrant the statement of certain general propositions. Chief among these are: passenger traffic, on the average, barely paid its way, and has fallen appreciably short at times; freight traffic, on the average, paid the net of

railway operation, but has recently yielded a relatively smaller surplus than before the War.

The use of more detailed data, and their analysis by more refined methods, would undoubtedly yield more precise results and would also admit of finding results of workable precision in respect of allocating costs as between different types of freight traffic and of allocating specific types of operating costs. All of the allocation equations may be regarded as statistical standards, giving the level of expenses to be expected on the average for given traffic densities. The use of these equations in determining rates, and in controlling the costs of operation of specific railways, may well result in more economical and more efficient conduct of the transportation industry as a whole.

The methods outlined are applicable also in any other industry where there is a large number of firms operating under roughly similar conditions and maintaining fairly detailed and substantially uniform accounts. For example, in the public utility field an allocation between electric power produced for residential consumption and for industrial purposes might be made. In department store retailing an allocation of costs as between major departments of the stores might conceivably be made. In the drug store business an allocation might be made between drug sales on the one hand and sales of confections and drinks on the other.

In agriculture and animal husbandry there are numerous cases where such an allocation might be possible if much more adequate records were kept than are available today. For example, an allocation of costs as between mutton and wool in the sheep raising industry should be possible. Moreover, in certain industries having alternative products the methods outlined above might be effective, for example, in the joint industry of corn raising, cattle feeding, and hog fattening.

In these and many other cases the allocation of costs, at least with a fair degree of approximation, by the methods outlined waits upon the collection of a sufficient body of adequately detailed statistical data. When the data do become available and the allocation equations are determined, problems of interpretation must be attacked. Moreover, in every case careful attention must be given to any peculiar circumstances which operate to limit the effectiveness and applicability of the method.

DISCUSSION BY M. O. LORENZ

This seems to be the first systematic attempt to separate railroad joint costs between freight and passenger services by the process of multiple correlation, although some experiments along this line have been made in the past. My impression is that this method is not so well adapted for solving this particular problem as is the customary method of analyzing the expenses by individual accounts and making a separation of expenses between services on some basis appropriate to each account. One major objection to Professor Crum's analysis is that he uses only three out of many important variables. He deals with ton-miles per mile of line, passenger-miles per mile of line, and expenses per mile of line. He omits average length of haul, average trainload, average carload, ratio of empty to loaded car-miles, proportions of various classes of freight traffic, ratio of commutation and regular passenger traffic, and the effect of grades. He thinks the omitted factors tend to cancel each other, but this is a dangerous assumption, especially as he deals with less than 200 railroads. If we knew nothing about railroad statistics except the three variables which he has used, his method might be valuable as giving some rough indication of freight and passenger unit costs, but when the expenses have been analyzed account by account, the separation of the major portion being without dispute, the rough method seems inappropriate. Exact mathematical computations may give only rough and even erroneous results if the conditions of the problem are too simply stated.

The conclusion that in all of the years studied the passenger service just about paid its way is at variance with the result of the method used by the Interstate Commerce Commission. The ratios of expenses to revenues for 1923 as published in the *Statistics of Railways* were as follows (in percentages):

Item	Eastern District	Southern District	Western District
Freight service	79.92	77.72	74.46
Passenger and allied services	81.05	78.15	85.21

It will be noted that in the above the passenger, mail, express and other passenger-train services are grouped together. It is not clear just how Professor Crum has separated the mail, express and other passenger-train expenses from the strictly passenger service.

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Freight service	79.92	77.72	74.46	77.55
Passenger and allied services	81.05	78.15	85.21	82.15

It will be noted that in the above the passenger, mail, exp[illegible] and other passenger-train services are grouped together. It is [illegible]clear just how Professor Crum has separated the mail, express [illegible]hen passenger-train expenses from the strictly passenger service [illegible]enses.

Professor Crum does not make it clear whether he regards any part

of the aggregate railroad expenses as "truly joint." His equation for 1923, $2022 = .0073F + .030P + 285.4$, seems to indicate that the 285.4, or 14 per cent of the total, remains unallocated, but he tells me that this constant is not in any way identifiable with the unallocated expenses. An interesting statistical exercise and test of his method would be to confine it to those expense accounts concerning the apportionment of which there is little dispute, such as the repair of equipment and the important transportation expenses.

One evident and distinct advantage of the Interstate Commerce Commission method over Professor Crum's method is the applicability of the former to one road or any number of roads to be included in a rate group.

The suggestion is made in the paper that the regression equation is an operating standard and might be of great usefulness in measuring efficiency. My impression is that the operating conditions obtaining on an individual railroad contain so many features peculiar to that road that no simple yard stick of comparative efficiency as between railroads can be applied for expenses as a whole.

THE CONSTRUCTION OF AN INDEX NUMBER OF BOND YIELDS IN THE UNITED STATES, 1859 TO 1926[1]

By Fred R. Macaulay, *National Bureau of Economic Research*

The National Bureau of Economic Research has for some time been engaged upon an investigation into the history of interest rates and bond yields in the United States, monthly since January 1859. The reasons for this particular date are somewhat accidental. Daily quotation sheets of the New York Stock Exchange are available from January 1, 1859, but we were unable to find any file going further back. We did not feel that it was particularly worth while struggling along into the earlier period without this source of material, as the magazine and newspaper reporting was irregular and inaccurate. Furthermore, the particular class of bonds which we have found most useful since 1859, namely railroad bonds, becomes less and less significant of any general investment condition as their quotations are pushed back of that date. Only a few years before, the railroads were in their mere infancy.

The index numbers for bond yields which are presented in this article are used to illustrate an attempted solution of the theoretical problem of measuring the yield of long-term loans of unchanging type with respect to such fundamental characteristics as maturity, security, marketability, etc. The bonds actually used were railroad bonds listed on the New York and Philadelphia Stock Exchanges. The reasons for using railroad bonds for such a purpose are numerous. There has been greater uniformity in the railroad bond market than in the markets for industrial or public utility bonds. The number of industrial and public utility bonds which would have been useful in constructing such an index number was almost negligible in the earlier period. Until very recent times nearly all industrial and public utility bonds have been of a lower grade than the better railroad bonds. In the later period a relatively large percentage of them are automatically eliminated because of such disturbing elements as convertibility, redeemability, etc. While quotations for municipal and state bonds go very far back, we found great difficulty in using much of this material. One peculiar reason was that in the earlier period it was often

[1] A paper read at the eighty-seventh Annual Meeting of the American Statistical Association, New York City, December 30, 1925.

impossible to identify the bonds from the printed descriptions in the price sheets. For example, for a number of years there were quotations on the Philadelphia Stock Exchange sheets for "Penna. 6's," though for most of the period we were unable to discover what particular bonds were referred to, there being at all times a number of 6's of various maturities outstanding.

Finally, though the movements of second or third grade railroad bonds in any particular period might imaginably be diametrically opposite to those of second or third grade bonds of another class such as industrials or public utilities, we would not expect this to be true of the very highest grade of bonds of any class. The factors influencing such bonds would be of a monetary, credit, financial, or even of a general business type, but not connected primarily with changes in the credit or other characteristics of the business issuing the bonds.[1] We felt therefore that if from an analysis of railroad bonds we could construct an index number showing the probable movement of the yields of the highest grade of railroad bonds, we would probably have an index of the movements of the yields of the highest grade of bonds in general, or at least, an index of railroad bond yields which would be only very negligibly affected by the changing status of the roads themselves.[2]

It is necessary at this point to draw attention to the fact that the whole concept of an index number showing the yield year by year of long-term loans of an unchanging character is somewhat vague not only from a statistical standpoint but also from the standpoint of economic theory. For what kind of a railroad bond in 1880 would the yield be strictly comparable with the yield of the very highest grade of railroad bond at the present time? We might, in 1880, have had quotations for a bond which, because of its issuing road and position among the securities of that road, could be considered as absolutely secure in any but a metaphysical sense. However, this would not solve such a problem as that arising from the relation of the amount of such type of bonds outstanding in 1880 compared with the demand for that particular type of bond in 1880, to the same consideration at the present time. For example, though we might consider Liberty Bonds as absolutely secure at the present time, if, as seems not unlikely, the amount of such bonds outstanding declines radically in the next ten years, their position in the general bond market will certainly be

[1] The movements of tax-exempt bonds in recent years have, of course, been somewhat different from the movements of taxed bonds.

[2] In the report which the National Bureau expects to publish we shall present not only index numbers derived from railroad bonds, but also index numbers derived from industrial, public utility, municipal and state bonds. Because of the circulation privilege, United States Government Bonds were, for most of the period, not comparable with other types.

changed. Though it would be difficult to say that their security would be appreciably greater than at the present time, they would undoubtedly sell on a lower yield basis with respect to the general mass of high grade bonds than they do at the present time. The markets for bonds of different types are to a considerable extent quite different markets.

The first task in connection with the construction of our index number was to discover a large number of railroad bonds having sufficiently numerous monthly quotations to be useful for our purpose. In this work we relied for the later periods upon the files of financial periodicals, more especially the *Commercial and Financial Chronicle.* For the period from January 1859 to January 1878 we used the original daily quotation sheets on file in the libraries of the New York and Philadelphia Stock Exchanges. In the early period we were faced with numerous difficulties connected with the necessary description of the bonds quoted. For example, before we became certain of the exact interest dates and the date of maturity of one bond, it was necessary for the Lehigh Valley Railroad to send a man up to the coal fields to unearth records which had not been examined for thirty years.[1] Having collected prices for an extremely large number of bonds, we weeded out those bonds which unmistakably were influenced much more by the changing fortunes of a particular road than they were by any general economic conditions. This was done by constructing tentative index numbers and comparing the yields of the individual bonds with such index numbers. When such comparison showed any sudden, violent, and unusual movement in the yield of a particular bond, some attempt was made to discover the peculiar conditions which caused such movement and to decide whether we should or should not eliminate such bond from our calculations for that period. This procedure seemed to us a reasonable one, particularly in view of the fact that we were not able to collect yields for a large enough number of bonds to give us any confidence that violent irregularities in individual issues would influence the general picture only to a microscopic degree.

The manner in which we computed the yields of the bonds from the prices may be briefly indicated. Before January 1909 bond prices on the New York Stock Exchange were quoted "flat" and not as they have been quoted since that time "and interest." The first step necessary to the computation of the yield of any particular bond for any particular month in the period before January 1909, was, therefore, the subtraction of the accrued interest to the middle of the month. After

[1] Lehigh Coal and Navigation 1st M. 6's, quarterly J. A. J. O., maturity March 1, 1870. We take this opportunity to thank the Railroad for its assistance in this matter.

obtaining thus a series of monthly "and interest" bond prices, the problem was to calculate yields with the minimum expenditure of time and money possible. To do so by the old fashioned arithmetic double interpolation for time and price in an old-style bond yield-table, having yield as the independent variable, would, with the tens of thousands of quotations which we were handling, have been very impractical. Our first improvement on this method was to chart actual prices and dates on cross-section paper and to draw on the same sheet the various mathematical yield lines. The necessary interpolations were then made by means of a graduated piece of celluloid. On the scale which we used the probable error involved in this procedure was calculated to be less than 1/100 of one per cent. However, shortly after the beginning of our work a brilliantly ingenious bond yield monograph was invented by D. C. Johnson and M. A. Darville.[1] We very carefully examined and checked this monograph and discovered that its probable error was, in general, less than 2/100 of one per cent. We realized this would not appreciably affect the accuracy of our final results. The use of this device reduced still further the time necessary for our computations.

Having computed a large number of monthly yield series for bonds of apparently a highly representative type and free from the more glaringly erratic movements which sometimes occur in individual issues, and whose maturity was at all times at least ten years distant, our problem was to decide upon the most satisfactory type of averaging. We immediately realized that a chain number was made absolutely necessary by the nature of our data. To simplify computations we introduced new bonds only in January and dropped old ones only in February so that we had always a homogeneous group from January to January of each year. We used no bond which was not available for at least six consecutive years. We did not feel any particular call to weight individual yields other than equally.[2] While it would have been possible to construct a set of arbitrary weights based upon imagined relative importance of the particular bonds, it would have been very laborious to have constructed any set of weights based upon actual measurements of number of bonds outstanding, number traded in, or any other such criterion. Moreover, we did not feel that any particularly strong defense could be made of any such prccedure after it had been done. Assuming, then, the weight of each yield to be at all times unity, we decided that a simple geometric average would best fulfill the

[1] Published by Prentice-Hall, Inc.

[2] The problem of weighting which is presented later in this paper and which is connected with the weighting of two variables before fitting a regression line is, of course, a different problem from the above.

various tests and especially, the so-called circular test.[1] The fulfilling of the circular test, of course, eliminates all purely mathematical drift in an upward or downward direction.

We next proceeded to calculate 67 individual geometric mean index numbers, each index number extending from one January to the next January. We then tied these 67 index numbers together into one long series by beginning with the geometric mean of all the bonds in January 1925 and working back by Januaries to January 1859. If at any January, say January 1924, we discovered that the index number which ran from January 1923 to January 1924 was, for example 11/10 of the January 1924 value in the January 1924–1925 index number, we then multiplied the 1923–1924 values by 10/11. Upon the completion of this composite index number, we were rather astonished to see that while its January 1925 value was 4.774 per cent (the geometric mean of the 41 bonds used in January 1925), its January 1879[2] value was 6.157 per cent, while the geometric mean of the 23 bonds used in that month was only 5.721 per cent, in spite of the fact that the 41 bonds used in January 1925 were undoubtedly of a much higher general grade than the 23 bonds used in January 1879. Indeed, of the 23 bonds used in January 1879, the yield of only one was as high as 6.157 per cent, the figure which had resulted from constructing an index number in the above manner. By January 1859 this drift had gone so far that while the index number figure was 8.954 per cent, the average of the 12 bonds used in that month was only 7.129 per cent, and the yield of the worst bond, that is, the one showing the highest yield was in that month only 7.92 per cent. Furthermore, the above comparison does not present the picture in as strong colors as it should be presented. It was necessary that we take poorer and poorer grades of bonds as we went back in time, if we were to have a large enough number in any one month to give us any confidence that we could smooth out the erratic movements of individual issues. There seems every reason for believing that the bonds which each month throughout the period showed the very lowest individual yields were more nearly of the same grade throughout the period than, for example, the bonds showing the highest yields or even the averages of all the bonds. Comparing our composite geometric chain index number with the individual bond showing the lowest yield in January 1925, we find the composite (4.774 per cent) to be 106.09 per cent of the yield (4.50 per cent) of the best bond used in that month; in January 1879 the composite index number

[1] If the weights are all equal, the simple geometric, which in Fisher's notation is No. 21, is identical with two of his "superlative" numbers, namely, No. 1323 and No. 5323.

[2] Resumption of Specie Payments.

(6.157 per cent) is 126.69 per cent of the yield (4.86 per cent) of the best bond used in that month; in January 1859 the composite index number (8.954 per cent) is 150.74 per cent of the yield (5.94 per cent) of the best bond used in that month. The arithmetic differences are, of course, still more startling than the ratios. They run January 1925, 0.274 per cent; January 1879, 1.297 per cent; January 1859, 3.014 per cent.

This drift of the geometric chain index number is not a technical or mathematical drift. It is not a "bias," to use Fisher's expression. The simple geometric average has no "bias." The drift is an economic drift. It is the result of improvement in the poorer bonds with the passage of time; second mortgages become first mortgages, and again with the growth of the railroad the percentage of its earnings necessary to cover all bonds down to and including the particular issue under discussion declines. The very nature of the bonds is changing. If then the downward drift be an economic and not a technical drift, it would of course be thoroughly indefensible to attempt to correct it by using some type of average having a technical or mathematical *upward* drift or "bias," such as a chain simple arithmetic average of relatives with, in each case, the first year as base.

After a considerable amount of work at this stage of the investigation, we came to the conclusion that the drift might be negligible in the very best of the bonds we had used. We then constructed an index number by taking each January the geometric mean of the yields of the 5 bonds having the lowest yields in that January, and the geometric mean of the yields of the same 5 bonds in the *succeeding* January. Chaining these index numbers together gave a curious result. The bonds were necessarily very high grade and the possibility of decline was evidently greater than the possibility of improvement. Tying up the resulting index number with the yield of the best 5 bonds in January 1925 (4.531 per cent) we got for January 1879, a yield of 4.020 per cent, and for January 1859 a yield of 4.516 per cent. Now these figures are probably as close to the yield of bonds which in 1879 and in 1859 would have been comparable with the best 5 in 1925 as the geometric index of all the bonds was in those earlier years to a group of bonds which in those years would have been comparable to the total group used in 1925. However this index of 5 bonds gives yields unmistakably too *low* for the earlier years. The ratio of this index number to the yield of the best bond used was, in 1925, 1.0069; in 1879, 0.8272; in 1859, 0.7603. Constructing another index number by taking each January the geometric mean of the yields of the 5 bonds having the lowest yields in that January and the geometric mean of the yields

of the same 5 bonds in the *preceding* January, we found our moderate upward drift changed into a pronounced downward drift. This index number showed for January 1925 a yield of 4.531 per cent, for January 1879 a yield of 6.823 per cent, and for January 1859 a yield of 11.220 per cent. The ratio of this index number to the yield of the best bond used was in January 1925, 1.0069; in January 1879, 1.4039; and in January 1859, 1.8889. A third procedure seemed more logical and gave a much more reasonable picture. Taking the geometric mean of the yield of the best 5 bonds in any January, and the geometric mean of the yields of the best 5 bonds in the succeeding January (whether these latter bonds were or were not the same bonds used in the preceding January, so long as both sets of 5 were selected from the same total group) the resulting index number gave, for January 1925, 4.531 per cent; for January 1879, 5.205 per cent; and for January 1859, 7.008 per cent. The ratio of the above figures to the yields of the best individual bonds was for January 1925, 1.0069; for January 1879, 1.0710; for January 1859, 1.1798. Though the downward drift in this result is much less than in the geometric index number for all the bonds, it has not been eliminated.

Another index number which we constructed illustrates well the fact that it is the higher yield, poorer bonds in our selection which have shown the pronounced decline in yield. This particular index number was constructed from the *worst* 5 bonds each January whether these bonds were or were not the same bonds as in the preceding January, so long as both sets of 5 were selected from the same total group. The results of this procedure showed for January 1925, 5.028 per cent; for January 1879, 7.301 per cent; and for January 1859, 11.561 per cent. The ratio of these figures to the corresponding figures for the similarly constructed index number of the *best* 5 bonds was for January 1925, 1.1097; for January 1879, 1.4027; for January 1859, 1.6497. Index numbers based upon the upper and the lower quartiles of the yields would, of course, tell a similar story to that just related, though their movements, with the number of bonds we were using, would be somewhat erratic.

If then, there be a downward drift in the yields of all these bonds, which, however, is much more pronounced in the yields of the higher yield bonds, and if our problem is that of investigating the history of long-term interest rates rather than the yields of groups of specific bonds, what should be done next? No kind of mere averaging which would be at all legitimate could solve our problem. It would be easy enough to take care of our troubles, if they were due, for example, to the higher yield bonds fluctuating more, but not when such bonds show

a drift in one direction. On the other hand, an examination of the movements of the very best bonds which we used had strongly suggested that there was relatively little systematic downward drift in this group. Our problem then resolved itself into discovering what would have been the yield of bonds of this grade each year—or perhaps we had better say the yield of bonds a shade better than this grade—if there had been available a large enough number of such bonds to construct any reasonable average. However, as there were almost no quotations for any such ultra high grade railroad bonds, the problem became one of estimating the movements of such a group from the material we actually had. We handled this problem with the data available by attempting to discover whether there were any mathematical relationship between the various yields in consecutive Januaries. We worked always with the logarithms of the yields. The reasons for doing so were numerous, and will be presented in detail in the final publication of this investigation.

Our first approach was to consider the logarithms of the yields of the *earlier* January as the *independent* variable and the logarithms of the yields of the *later* January as the *dependent* variable. We then began fitting, by the method of least squares, a series of regression lines to the material for successive years, quickly realizing, however, before proceeding far with the work, that this would give us a series of index numbers spreading out in an absolutely uncanny manner as we went back in time. To pick a yield in January 1859 which under this procedure would by January 1925 not be higher than the average of the best 5 bonds, meant using for 1859 a yield which was an almost infinitesimal fraction of one per cent. Using the *later* January as the *independent* variable the results were also absurd, though of course, in the opposite direction. Moreover, as we considered the matter further, we felt strongly the lack of any reason for either of the above procedures. Neither the earlier nor the later January has any peculiar right to be considered the independent variable in a problem of this kind. To borrow an expression from the theory of least squares, both variables are tinged with error. Whatever procedure be used this fact must be fully recognized. Under the doctrine of least squares what was demanded was not that the sum of the squares of the vertical deviations of each datum value from its theoretical value should be a minimum, or that the corresponding sum of the squares of the horizontal deviations should be a minimum, but rather that the sum of the squares of the vertical *and* the horizontal deviations, or perhaps we had better say displacements, of each datum value from its theoretical value should be a minimum. We therefore fitted straight lines to each pair of

Januaries in such a manner that the sum of the squares of the vertical *and* the horizontal displacements of each datum point from the corresponding theoretical point on the straight line would be a minimum.[1]

In handling the problem in this manner we were immediately faced with the question of what weights to give the observations in one January, as compared with those in the next January. At first glance it would seem simple and proper to give the logarithms of the yields in each January equal weights. But the concept of "equal weights" is in a case like this a highly dangerous one, for it often amounts merely to equal weighting of the variables when expressed in the accidental scales of the particular problem. It shows itself most clearly where the two variables are measured in absolutely unlike units such as the height and the weight of men. When, as in our problem, both variables are expressed in percentages, there is a tendency to let mere words lull us into the assumption that we already have something in the nature of equal weights.[2] However, in our particular problem, this procedure would have meant treating a one per cent deviation of observation from theory in one January as of the same importance as a one per cent deviation in the succeeding January, in spite of the fact that the scatter in the later January might be so much greater than in the earlier January that its standard deviation would be double that of the first January. Theoretical considerations led to the conclusion that the most logical treatment was to weight the observations in inverse proportion to the squares of their standard deviations. This amounts to saying that our results would then be unaffected by the nominal scales used in our scatter diagrams. Actual application of this method of treatment supported strongly our theoretical reasoning. Computation showed that the naive so-called "equal weighting" gave us an index number for the best grade of bonds which was unmistakably too high in the early period or too low in the later period.

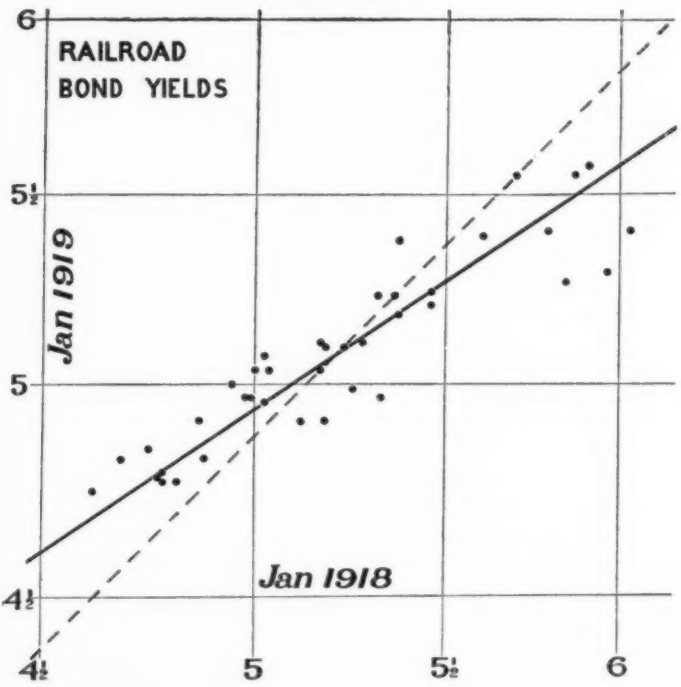
Constructing then an index number based upon regression lines fitted on the assumption that both variables were tinged with "error," and weighting the variables in inverse proportion to the squares of their standard deviations, we had for each year a straight line going through the mean of the system and having a slope equal to $\frac{\sigma_y}{\sigma_x}$, where σ_y is the standard deviation of the later January and σ_x is the standard deviation of the earlier January. Let me illustrate some results of such

[1] Compare Merriman: "The Determination by the Method of Least Squares of the relation between two variables connected by the equation $Y=AX+B$, both variables being liable to errors of observation." Report United States Coast and Geodetic Survey 1890, p. 687.

[2] Merriman does not examine this statistical pitfall.

a procedure. The bond having the lowest yield in January 1859 was a Pennsylvania R. R. 6 per cent first mortgage bond, issued October 14, 1852 and maturing December 31, 1880. This was an extremely high grade bond, actively traded in, and having in January 1859 a reasonably long time to maturity—22 years. Taking the average yield of this bond in January 1859 (5.94 per cent) as a starting point and applying the yearly equations derived from the total of all the bonds, we obtain an index number which is shown on Chart III, as the

CHART I

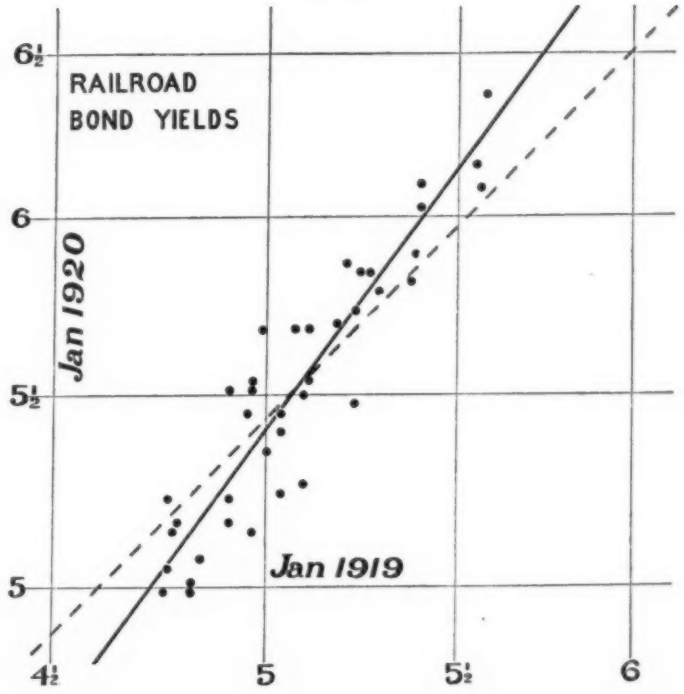


lowest index number on the chart (Line D). This seems a very excellent result. While generally below the yield of the best bond in any particular year, this index is not always so, and in periods where we had the very highest grade of bonds the yields of such bonds are often actually lower than this index number. The ratio of this index number to the best bond was in January 1879, 0.9891 and in January 1925, 0.9949. Its general behavior and the confidence we felt in its theoretical foundation made us decide not to attempt further refinements in method, though a number of considerations which will be discussed in the final report of this investigation make us suspect that the truth lies

in an even more radical procedure than beginning with the yield of this Pennsylvania bond. The probability is that even this index number (Line D) is too high in the earlier years. We believe that, if such be the case, it is the result of overestimating the quality of this early Pennsylvania bond, and not the result of the theory involved in the method of constructing the index number.

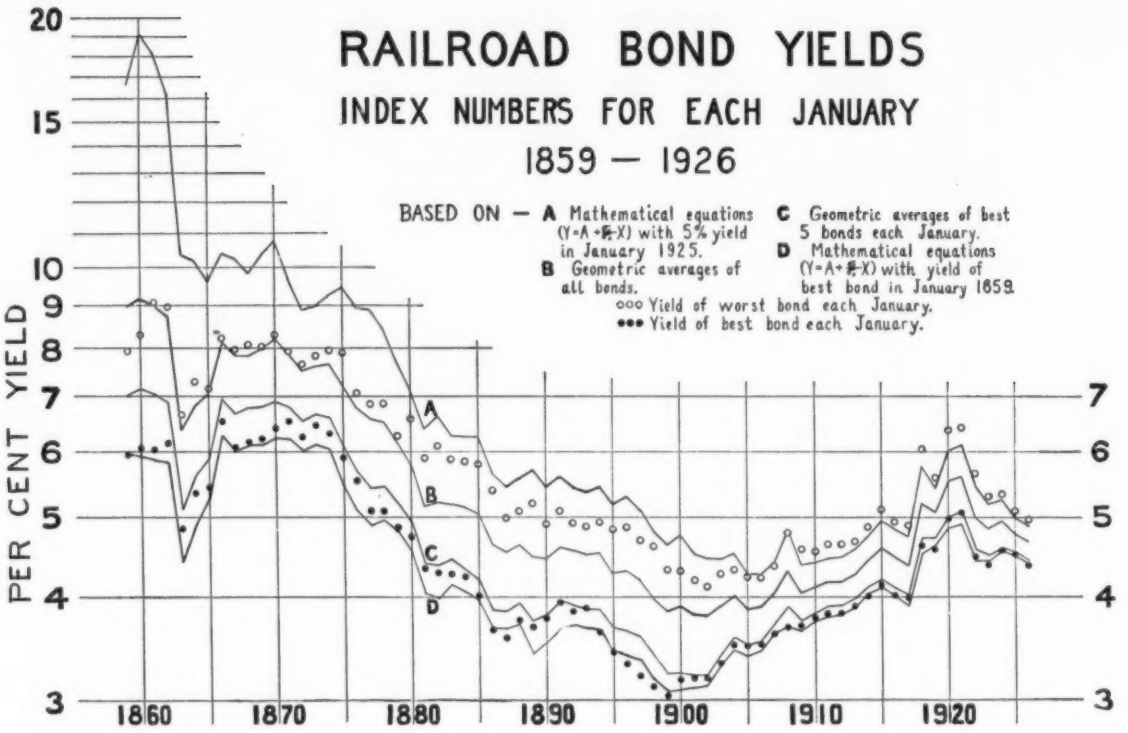
The three charts accompanying this article illustrate the method and its results. Charts I and II show scatter diagrams for the years

CHART II



January 1918 to January 1919, and January 1919 to January 1920. These charts are drawn on double logarithmic scales. Each black circle represents one bond used in the index number for this period. On each chart the 45° line goes through the mean of the system. Making a reading for the earlier date (X axis) and erecting a perpendicular up to the 45° line, we may read on the Y axis the corresponding yield for the later date, if we assume that no matter what the yield of a bond for the earlier date its percentage change in yield from the earlier to the later date would be the same. In other words, the 45° line corresponds to

CHART III


RAILROAD BOND YIELDS
INDEX NUMBERS FOR EACH JANUARY
1859 — 1926
BASED ON — A Mathematical equations (Y=A+BX) with 5% yield in January 1925.
B Geometric averages of all bonds.
C Geometric averages of best 5 bonds each January.
D Mathematical equations (Y=A+BX) with yield of best bond in January 1859.
ooo Yield of worst bond each January.
••• Yield of best bond each January.
PER CENT YIELD
20
15
10
9
8
7
6
5
4
3
A
B
C
D
1860
1870
1880
1890
1900
1910
1920

the assumptions involved in constructing index numbers from simple geometric means of all the bonds. The other line, which in January 1918 to January 1919 is less steeply inclined and in January 1919 to January 1920 more steeply inclined than 45°, is in each case a regression line through the material so placed that the sum of the squares of the vertical *and* horizontal "displacements" shall be a minimum. It is immediately seen that these latter lines describe the data much more adequately than the 45° lines. In connection with Chart III, attention should be drawn to the fact that the drift in the geometric index of all the bonds is by no means uniform. In certain periods it has shown no systematic drift; for example, the movement of the theoretical indexes from January 1883 to January 1898, and from January 1911 to January 1913 were practically the same as the movements of the simple geometric index of all the bonds. However, in some periods the drift was very pronounced; for example, January 1859 to January 1873, January 1900 to January 1907, January 1908 to January 1911, and January 1920 to date. The reader should note that Chart III is drawn on a logarithmic scale. If the chart had been drawn on a natural scale the drift would have appeared even more pronounced.

The yields discussed in this article are all *paper* yields. A dollar has been the unit whether that dollar was or was not worth a particular weight of gold. In the final report of this study *gold* yields for each of these bonds for each separate month will be given for the period January 1, 1862 to January 1, 1879. Such yields show a great rise in the early sixties instead of the great fall shown in the chart for paper yields. Tables showing the *paper* yield of each individual bond for each separate month from January 1859 to January 1926 will also be given. In the same report all index numbers will be presented for each quarter of the year and some of the numbers will be given for each month from January 1859 to date.

Finally, I wish to take this opportunity to acknowledge with the most sincere thanks the help which has been so generously given by so many persons and institutions in connection with this study. To mention only three, the New York Stock Exchange and the Philadelphia Stock Exchange were courteous and helpful to the last degree, in each case assigning special space for our workers. The Metropolitan Life Insurance Company went so far as to put at our disposal a number of statistical clerks for a very considerable time.

THE RELIABILITY OF A PERCENTAGE

By Hornell Hart, *Bryn Mawr College*

The orthodox formula is unsatisfactory for analyzing differences between percentages of items having a given characteristic in two samples. That formula, as stated by Yule [1] is as follows:

$$\epsilon^2_{12} = p_0 q_0 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)$$

where ϵ_{12} is the standard error of the difference between the two percentages, p_0 is the weighted average of the percentages, q_0 is one minus p_0, and n_1 and n_2 are respectively the numbers of items in the samples considered.

A specific case will illustrate limitations in the application of this formula. In a certain [2] rural survey 437 children 10 years of age and over were classified according to whether or not the children and their parents were church members. Of 51 children with neither parent a church member none were themselves members. Of 64 children, each of whom had one parent a member and one parent not a member, 45.3 per cent were themselves church members. The other 322 children had both parents members, and of these children 88.5 per cent were members. Consider the question of the significance of the difference between the percentages of church membership among children with neither parent a church member and among children with one parent a member, i. e., between zero per cent in a sample of 51 items and 45.3 per cent of 64 items. Applying Yule's formula, the standard error of the difference is found to be 8.1 per cent. The observed difference is 45.3 per cent, or 5.6 times the standard error.

Assuming that the above groups of items fulfill the conditions of simple sampling, it is safe to conclude from the ratio between the observed difference and its standard error, that the probability is negligible that the two samples were really drawn from universes both having 25.2 per cent of church members among the children. While this gives a fairly conclusive answer to the question as to whether the whole of the observed difference is due to chance, it does not tell what the statistician really wishes to know. What is needed is a

[1] G. Udny Yule, *Theory of Statistics*, London, 1919, p. 269.

[2] *A Rural Social Survey of Orange Township, Blackhawk County, Iowa;* Iowa State College Agricultural Experiment Station, Bulletin No. 184, 1918.

basis for determining between what limits it is safe to say that the difference lies between the percentages of church membership of children with neither parent a member and of children with one parent a member, in groups of children comparable with the samples gathered. The orthodox formula gives a basis for concluding that the samples were *not* drawn from a certain sort of undifferentiated universe. What is needed is a basis for determining, within limits, from what type of universes the samples *were* drawn. The needed type of answer would be something as follows:

These 51 children with neither parent a church member represent a universe with respect to which the probability is .99 that between 0 and 8 per cent of the children are church members; the 64 children with one parent a member represent a universe with respect to which the probability is .99 that 30 to 60 per cent of the children are members. Granted the accuracy of the enumeration and tabulation and the impartiality and accuracy with which the definitions have been applied, and assuming that each child in the samples is an independent item, it is safe to conclude that among children like those in the sample with one parent a church member the percentage of children who are church members is not less than 22 per cent greater and not more than 60 per cent greater than among children with neither parent a church member.

No basis is provided for drawing such conclusions in the procedure outlined by Yule. A development of the theory of probability with a view to meeting this need is the purpose of the present paper.

Suppose that 100 urns containing black and white balls were maintained so that the proportion of black balls in the first urn was .995, in the second .985, in the third .975, and so on by steps of .010 until the last urn contained a proportion of .005. Suppose that from each urn 100,000 samples were drawn, each sample containing n balls, and that each of these samples was ticketed to indicate from what urn it came. Suppose that all samples containing pn black balls were collected, and formed into a frequency series according to the proportions of black balls in the urns of their origins. If from such a series 99 per cent of the samples were selected in such a way as to come from the smallest possible number of different urns, what would be the largest and the smallest proportions of black balls in the urns so represented? The answer to this question gives an approximation to the information which it is desired to have relative to the reliability of a percentage. The question may be stated in general terms as follows: if in a sample of n items a proportion p shows a given characteristic, and if there is no reason, other than the sample in hand, to assume that any particular

proportion of items having that characteristic is more likely than any other proportion in the universe from which the sample is drawn, what is the shortest range within which the probability is .99 that the proportion of such items lies in the universe from which the sample is drawn?

To avoid confusion, the proportion of black balls in the samples will be referred to as small p, the proportion of white balls in the samples as small q, the proportion of black balls in the urn under consideration will be referred to as large P and the proportion of white balls in the urn as large Q. From the theory of probability it follows that, if 100,000 samples, each containing n items, are drawn from

TABLE I—THE RELIABILITY OF A

If a sample contains the number of items shown at the left hand end of any row, and has the percentage of items in the universe which this sample represents lies between p minus the first number given at the intersection of that row the evidence furnished by the sample in question, any percentage is as likely as any other to occur in the universe

n \ p	0	1	3	5	7	10	15	20	25	30	35	40	45
10	−0, 35	−1, 35	−3, 36	−5, 36	−7, 36	−10, 37	−14, 37	−18, 38	−21, 38	−24, 37	−27, 37	−29, 36	−31, 35
11	−0, 32	−1, 33	−3, 33	−5, 34	−7, 34	−10, 35	−14, 35	−18, 36	−21, 36	−24, 36	−26, 36	−28, 35	−30, 34
12	−0, 30	−1, 31	−3, 31	−5, 32	−7, 32	−10, 33	−14, 34	−17, 34	−20, 35	−23, 35	−25, 34	−28, 34	−29, 33
13	−0, 28	−1, 29	−3, 29	−5, 30	−7, 31	−10, 32	−13, 32	−17, 33	−20, 34	−23, 34	−24, 33	−27, 33	−29, 32
14	−0, 27	−1, 27	−3, 28	−5, 29	−7, 29	−10, 30	−13, 31	−16, 32	−19, 33	−22, 33	−24, 32	−26, 32	−28, 31
15	−0, 25	−1, 26	−3, 27	−5, 27	−7, 28	−10, 29	−13, 30	−16, 31	−19, 32	−22, 32	−23, 32	−26, 31	−27, 30
16	−0, 24	−1, 25	−3, 25	−5, 26	−7, 27	−9, 28	−13, 29	−16, 30	−19, 31	−21, 31	−23, 31	−25, 30	−27, 29
17	−0, 23	−1, 24	−3, 24	−5, 25	−7, 26	−9, 27	−13, 28	−15, 29	−18, 30	−21, 30	−23, 30	−25, 29	−26, 29
18	−0, 22	−1, 23	−3, 23	−5, 24	−7, 25	−9, 26	−12, 27	−15, 28	−18, 29	−20, 29	−22, 29	−24, 29	−25, 28
19	−0, 21	−1, 21	−3, 22	−5, 23	−7, 24	−9, 25	−12, 27	−15, 27	−18, 28	−20, 28	−22, 28	−24, 28	−25, 27
20	−0, 20	−1, 20	−3, 21	−5, 22	−7, 23	−9, 25	−12, 26	−15, 27	−18, 27	−20, 28	−21, 28	−23, 27	−24, 27
21	−0, 19	−1, 20	−3, 21	−5, 22	−7, 23	−9, 24	−12, 25	−14, 26	−17, 27	−19, 27	−21, 27	−23, 27	−24, 26
22	−0, 18	−1, 19	−3, 20	−5, 21	−7, 22	−9, 23	−12, 25	−14, 25	−17, 26	−19, 26	−21, 26	−23, 26	−24, 26
23	−0, 18	−1, 18	−3, 20	−5, 20	−7, 22	−9, 23	−12, 24	−14, 25	−17, 25	−19, 26	−20, 26	−22, 26	−23, 25
24	−0, 17	−1, 18	−3, 19	−5, 20	−7, 21	−9, 22	−12, 24	−14, 24	−17, 25	−19, 25	−20, 25	−22, 25	−23, 25
25–29	−0, 15	−1, 16	−3, 17	−5, 18	−7, 20	−9, 21	−11, 22	−14, 23	−16, 23	−18, 24	−19, 24	−21, 24	−22, 24
30–34	−0, 13	−1, 14	−3, 15	−5, 16	−7, 18	−8, 19	−11, 20	−13, 21	−15, 22	−17, 22	−18, 22	−20, 22	−20, 22
35–39	−0, 12	−1, 12	−3, 14	−5, 15	−7, 16	−8, 17	−11, 19	−13, 19	−14, 20	−16, 20	−17, 21	−18, 21	−19, 20
40–44	−0, 10	−1, 11	−3, 13	−5, 13	−7, 14	−8, 16	−11, 17	−12, 18	−14, 19	−15, 19	−16, 19	−17, 19	−18, 19
45–49	−0, 9	−1, 10	−3, 11	−5, 13	−7, 14	−8, 15	−10, 16	−12, 17	−13, 18	−15, 18	−16, 18	−17, 18	−17, 18
50–59	−0, 8	−1, 9	−3, 10	−5, 11	−6, 12	−8, 13	−10, 15	−11, 16	−13, 16	−14, 17	−15, 17	−16, 17	−16, 17
60–69	−0, 7	−1, 8	−3, 9	−5, 10	−6, 11	−7, 12	−9, 13	−11, 14	−12, 15	−13, 15	−14, 16	−14, 16	−15, 15
70–79	−0, 6	−1, 7	−3, 8	−5, 9	−6, 10	−7, 11	−9, 12	−10, 13	−12, 14	−12, 14	−13, 14	−14, 15	−14, 15
80–89	−0, 5	−1, 6	−3, 7	−5, 8	−6, 9	−7, 10	−9, 11	−10, 12	−11, 13	−12, 14	−12, 14	−13, 14	−14, 14
90–99	−0, 5	−1, 5	−3, 7	−5, 8	−6, 8	−6, 10	−8, 11	−9, 12	−11, 12	−11, 13	−12, 13	−12, 13	−13, 13
100–119	−0, 4	−1, 5	−3, 6	−4, 7	−5, 8	−6, 9	−8, 10	−9, 11	−10, 11	−11, 12	−11, 12	−12, 12	−12, 12
120–139	−0, 4	−1, 4	−3, 5	−4, 6	−5, 7	−6, 8	−7, 9	−8, 10	−9, 10	−10, 11	−10, 11	−11, 11	−11, 11
140–159	−0, 3	−1, 4	−3, 5	−3, 5	−4, 6	−5, 7	−6, 8	−8, 9	−8, 9	−9, 10	−10, 10	−10, 10	−10, 10
160–179	−0, 3	−1, 3	−2, 4	−3, 5	−4, 6	−5, 7	−6, 8	−7, 9	−8, 9	−9, 9	−9, 10	−10, 10	−10, 10
180–199	−0, 3	−1, 3	−2, 4	−3, 5	−4, 5	−5, 6	−6, 7	−7, 8	−8, 8	−8, 9	−9, 9	−9, 9	−9, 9
200–299	−0, 2	−1, 2	−2, 3	−3, 4	−3, 4	−4, 6	−5, 6	−6, 7	−7, 7	−7, 8	−8, 8	−8, 8	−8, 8
300–399	−0, 2	−1, 2	−2, 3	−2, 3	−3, 4	−4, 5	−4, 5	−5, 6	−6, 6	−6, 7	−6, 7	−7, 7	−7, 7
400–499	−0, 1	−1, 2	−1, 2	−2, 3	−3, 3	−3, 4	−4, 5	−5, 5	−5, 5	−5, 6	−6, 6	−6, 6	−6, 6
500–599	−0, 1	−1, 2	−1, 2	−2, 2	−2, 3	−3, 4	−4, 4	−4, 5	−5, 5	−5, 5	−5, 5	−5, 5	−5, 5
600–999	−0, 1	−0, 1	−1, 2	−1, 2	−2, 2	−2, 3	−3, 3	−3, 4	−4, 4	−4, 4	−4, 5	−4, 5	−5, 5
1000–1999	−0, 1	−0, 1	−1, 1	−1, 1	−1, 2	−2, 2	−2, 2	−3, 3	−3, 3	−3, 3	−3, 3	−3, 3	−3, 3
2000–2999	−0, 1	−0, 1	−1, 1	−1, 1	−1, 1	−1, 1	−2, 2	−2, 2	−2, 2	−3, 3	−3, 3	−3, 3	−3, 3
3000–3999	−0, 0	−0, 1	−0, 1	−1, 1	−1, 1	−1, 1	−1, 1	−2, 2	−2, 2	−2, 2	−2, 2	−2, 2	−2, 2
4000–4999	−0, 0	−0, 1	−0, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−2, 2	−2, 2	−2, 2	−2, 2	−2, 2
5000–9999	−0, 0	−0, 0	−0, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−2, 2

an urn maintained at a proportion of P black balls, the number of samples with pn black balls will, in accordance with the binomial expansion, be

$$\frac{100{,}000\ n!}{(qn)!\,(pn)!}P^{qn}Q^{pn}$$

Since it is the percentage distribution of the samples having pn black balls that is to be derived, and since the number of such samples from each urn has as a factor the same binomial coefficient, this coefficient may be eliminated from the calculations, and the problem reduced to finding the value of $P^{qn}Q^{pn}$ for all of the 100 values of P and Q, for

PERCENTAGE p IN A SAMPLE n

of a given type indicated at the top of any column, the probability is .99 that the percentage of items of the given type and column and p plus the second number given at that intersection, provided that it be assumed that, aside from represented.

50	55	60	65	70	75	80	85	90	93	95	97	99	100
−33,33	−35,31	−36,29	−37,27	−37,24	−38,21	−38,18	−37,14	−37,10	−36,7	−36,5	−36,3	−35,1	−35,0
−32,32	−34,30	−35,28	−36,26	−36,24	−36,21	−36,18	−35,14	−35,10	−34,7	−34,5	−33,3	−33,1	−32,0
−31,31	−33,29	−34,28	−34,25	−35,22	−35,20	−31,17	−34,14	−33,10	−33,7	−32,5	−31,3	−31,1	−30,0
−30,30	−32,29	−33,27	−33,24	−34,23	−34,20	−33,17	−32,13	−32,10	−31,7	−30,5	−29,3	−29,1	−28,0
−29,29	−31,28	−32,26	−32,24	−33,22	−33,19	−32,16	−31,13	−30,10	−29,7	−29,5	−28,3	−27,1	−27,0
−29,29	−30,27	−31,26	−32,23	−32,22	−32,19	−31,16	−30,13	−29,10	−28,7	−27,5	−27,3	−26,1	−25,0
−28,28	−29,27	−30,25	−31,23	−31,21	−31,19	−30,16	−29,13	−28, 9	−27,7	−26,5	−25,3	−25,1	−24,0
−27,27	−29,26	−29,25	−30,23	−30,21	−30,18	−29,15	−28,13	−27, 9	−26,7	−25,5	−24,3	−24,1	−23,0
−27,27	−28,25	−29,24	−29,22	−29,20	−29,18	−28,15	−27,12	−26, 9	−25,7	−24,5	−23,3	−23,1	−22,0
−26,26	−27,25	−28,24	−28,22	−28,20	−28,18	−27,15	−27,12	−25, 9	−24,7	−23,5	−22,3	−21,1	−21,0
−26,26	−27,24	−27,23	−28,21	−28,20	−27,18	−27,15	−26,12	−25, 9	−23,7	−22,5	−21,3	−20,1	−20,0
−25,25	−26,24	−27,23	−27,21	−27,19	−27,17	−26,14	−25,12	−24, 9	−23,7	−22,5	−21,3	−20,1	−19,0
−25,25	−26,24	−26,23	−26,21	−26,19	−26,17	−25,14	−25,12	−23, 9	−22,7	−21,5	−20,3	−19,1	−18,0
−24,24	−25,23	−26,22	−26,20	−26,19	−25,17	−25,14	−24,12	−23, 9	−22,7	−20,5	−20,3	−18,1	−18,0
−24,24	−25,23	−25,22	−25,20	−25,19	−25,17	−24,14	−24,12	−22, 9	−21,7	−20,5	−19,3	−18,1	−17,0
−23,23	−24,22	−24,21	−24,19	−24,18	−23,16	−23,14	−22,11	−21, 9	−20,7	−18,5	−17,3	−16,1	−15,0
−21,21	−22,20	−22,20	−22,18	−22,17	−22,15	−21,13	−20,11	−19, 8	−18,7	−16,5	−15,3	−14,1	−13,0
−20,20	−20,19	−21,18	−21,17	−20,16	−20,14	−19,13	−19,11	−17, 8	−16,7	−15,5	−14,3	−12,1	−12,0
−19,19	−19,18	−19,17	−19,16	−19,15	−19,14	−18,12	−17,11	−16, 8	−14,7	−13,5	−13,3	−11,1	−10,0
−18,18	−18,17	−18,17	−18,16	−18,15	−18,13	−17,12	−16,10	−15, 8	−14,7	−13,5	−11,3	−10,1	−9,0
−16,16	−17,16	−17,16	−17,15	−17,14	−16,13	−16,11	−15,10	−13, 8	−12,6	−11,5	−10,3	−9,1	−8,0
−15,15	−15,15	−16,14	−16,14	−15,13	−15,12	−14,11	−13, 9	−12, 7	−11,6	−10,5	−9,3	−8,1	−7,0
−14,14	−15,14	−15,14	−14,13	−14,12	−14,12	−13,10	−12, 9	−11, 7	−10,6	−9,5	−8,3	−7,1	−6,0
−14,14	−14,14	−14,13	−14,12	−14,12	−13,11	−12,10	−11, 9	−10, 7	−9,6	−8,5	−7,3	−6,1	−5,0
−13,13	−13,13	−13,12	−13,12	−13,11	−12,11	−12, 9	−11, 8	−10, 6	−8,6	−8,5	−7,3	−5,1	−5,0
−12,12	−12,12	−12,12	−12,11	−12,11	−11,10	−11, 9	−10, 8	−9, 6	−8,5	−7,4	−6,3	−5,1	−4,0
−11,11	−11,11	−11,11	−11,10	−11,10	−10, 9	−10, 8	−9, 7	−8, 6	−7,5	−6,4	−5,3	−4,1	−4,0
−10,10	−10,10	−10,10	−10,10	−10, 9	−9, 8	−9, 8	−8, 6	−7, 5	−6,4	−5,3	−5,3	−4,1	−3,0
−10,10	−10,10	−10,10	−10, 9	−9, 9	−9, 8	−9, 7	−8, 6	−7, 5	−6,4	−5,3	−4,2	−3,1	−3,0
−9, 9	−9, 9	−9, 9	−9, 9	−9, 8	−8, 8	−8, 7	−7, 6	−6, 5	−5,4	−5,3	−4,2	−3,1	−3,0
−8, 8	−8, 8	−8, 8	−8, 8	−8, 7	−7, 7	−7, 6	−6, 5	−6, 4	−4,3	−4,3	−3,2	−2,1	−2,0
−7, 7	−7, 7	−7, 7	−7, 6	−7, 6	−6, 6	−6, 5	−5, 4	−5, 4	−4,3	−3,2	−3,2	−2,1	−2,0
−6, 6	−6, 6	−6, 6	−6, 6	−6, 5	−5, 5	−5, 5	−5, 4	−4, 3	−3,3	−3,2	−2,1	−2,1	−1,0
−5, 5	−5, 5	−5, 5	−5, 5	−5, 5	−5, 5	−5, 4	−4, 4	−4, 3	−3,2	−2,2	−2,1	−2,1	−1,0
−5, 5	−5, 5	−5, 4	−5, 4	−4, 4	−4, 4	−4, 3	−3, 3	−3, 2	−2,2	−2,1	−2,1	−1,0	−1,0
−3, 3	−3, 3	−3, 3	−3, 3	−3, 3	−3, 3	−3, 3	−2, 2	−2, 2	−2,1	−1,1	−1,1	−1,0	−1,0
−3, 3	−3, 3	−3, 3	−3, 3	−3, 3	−2, 2	−2, 2	−2, 2	−1, 1	−1,1	−1,1	−1,1	−1,0	−1,0
−2, 2	−2, 2	−2, 2	−2, 2	−2, 2	−2, 2	−2, 2	−1, 1	−1, 1	−1,1	−1,1	−1,0	−1,0	−0,0
−2, 2	−2, 2	−2, 2	−2, 2	−2, 2	−2, 2	−1, 1	−1, 1	−1, 1	−1,1	−1,1	−1,0	−1,0	−0,0
−2, 2	−2, 2	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1, 1	−1,1	−1,1	−1,0	−0,0	−0,0

each assigned value of p and n. The writer has calculated, with assistance from Miss Julia Snell, the shortest .99 range for the following values of p: .0, .1, .3, .4, .6, .7, .9 and 1.0, in combination with the following values of n: 10, 20, 40, 80, 160, 320, 640, 1280, 2560, 5120, and 10,240. From the values so ascertained curves have been drawn, and from these curves the values given in Table I have been derived.

The illustration given above involving church membership was worked out by means of the table. Another illustration is afforded by data in a thesis [1] in which 47 delinquent boys in Chicago were compared

TABLE II

FATHERS' EARNINGS AND PRENATAL CARE OF MOTHERS FOR BABIES BORN IN BALTIMORE IN 1915 AS REPORTED IN THE STUDY OF INFANT MORTALITY IN THAT CITY BY THE UNITED STATES CHILDREN'S BUREAU, p. 205

Earnings of father	Number of mothers	Per cent without prenatal care	.99 range for per cents
0	232	41.4	33 to 49
Under $450	1,668	53.0	50 to 56
$450–549	1,551	58.7	56 to 62
550–649	1,566	59.8	57 to 63
650–849	2,546	52.7	50 to 56
850–1,049	1,675	40.5	37 to 44
1,050–1,249	696	30.0	26 to 34
1,250–1,449	444	32.9	27 to 39
1,450–1,849	390	21.3	16 to 27
1,850–2,249	143	18.2	10 to 27
2,250–2,849	101	13.9	5 to 24
Over 2,849	212	10.8	7 to 17
Not reported	239		
Total	11,463	47.5	46.3 to 48.7*

* The larger n becomes the nearer do the results obtained by Yule's formula approximate those obtained by the method outlined in this article. For values of n over 10,000 the use of the formula is recommended. The range for the total in Table II was obtained from the formula, multiplying the standard error by 2.58.

with 47 non-delinquent boys of comparable ages from the same area. Of the delinquent boys, 42.6 per cent are reported to have claimed to use gymnasiums or playgrounds, while of the non-delinquents only 19.2 per cent are reported to have made such a claim. Looking up these values in Table I it appears that the sample of delinquent boys represents a universe in which between 26 and 61 per cent claim to use gymnasiums or playgrounds, while the sample of non-delinquents represents a universe in which from 7 to 36 per cent make such claims. The samples are not large enough to prove definitely which group claims to use these recreational facilities more frequently, but it does

[1] Roy David Young, *Delinquency Among Boys and Uncontrolled Time*, M.A. thesis, Northwestern University.

CHART I

PERCENTAGES OF MOTHERS OF BABIES BORN IN BALTIMORE IN 1915 AT VARIOUS INCOME LEVELS WHO HAD NO PRENATAL CARE

indicate that, unless errors have been made in the study in question, the excess of use by non-delinquents cannot be expected to be over 10 per cent, while the excess of use by delinquents may be over 50 per cent.

A more elaborate application of this method is possible in a problem of the type illustrated in Table II and Chart I. The percentages of mothers without prenatal care are shown for the various income groups by the polygon in the chart. The limits within which the percentages are almost certain to lie in the universes represented are indicated by arrow heads. The best regression line is the one which best fulfills the two conditions of being as simple a curve as possible and of staying as near as possible to the center of the zone indicated by the arrow heads. It is clear that the curve reaches a maximum at some point in the neighborhood of $600 and that it is lower at incomes over $2750 than for any income group under $1250. The higher percentage in the group $1250–1449 than in the group $1050–1249 may be interpreted as due to accidental variation, or it may represent a real plateau in the regression line. The general shape of the curve is quite definitely determined.

A similar method of plotting regression lines may be used with averages. In this case, however, the zone within which the probability is .99 that the true average lies for the universe represented by a sample, is between the mean plus and minus $2.58\frac{\sigma}{\sqrt{n}}$, where σ is the standard deviation of the series, and n is the number of items in the sample.

PRICE MAKING FORCES IN COTTON MARKETS

By Hugh B. and Lucy W. Killough, *Brown University*

The price of cotton is in a continuous state of fluctuation. The trader pieces together bits of information about supply and demand, and compares the guesses of other traders before he backs his own estimate by a promise to buy or to sell. How is he to know which factors are the most important in determining the final price? In this paper several price making forces in cotton markets are analyzed and their relative importance measured.

American cotton has a world market and a world price. Except for the erratic period during the years of war and post-war liquidation, monthly prices of American middling cotton at New Orleans, New York, and Liverpool, and monthly prices of cotton on farms in the United States bore a close relation to each other. New Orleans and New York prices are to be expected to range above farm prices because of marketing costs. Liverpool prices, likewise, are to be expected to range above those of New Orleans and New York. The uniformity of month to month movements of American cotton prices in different markets gives universal application to a study based on one series of monthly average prices.

From 1915 to 1920 changes in cotton prices were abnormal. During this period annual price changes and short time oscillations were greater than before 1915 or since 1921. Government price fixing, obstruction to ocean-going freights, the closing of the New York Cotton Exchange, inflation, and war-time uncertainties prevented the quick price adjustments that usually occur between foreign and domestic markets. For these reasons the years from 1915 to 1920 inclusive are not included in subsequent comparisons.

At least three advance indicators of raw cotton prices are generally recognized. They are (1) the estimates of dealers as shown by futures prices; (2) the supply of cotton; and (3) evidences of the demand for cotton embodied in indices of cloth prices and movements of raw cotton.

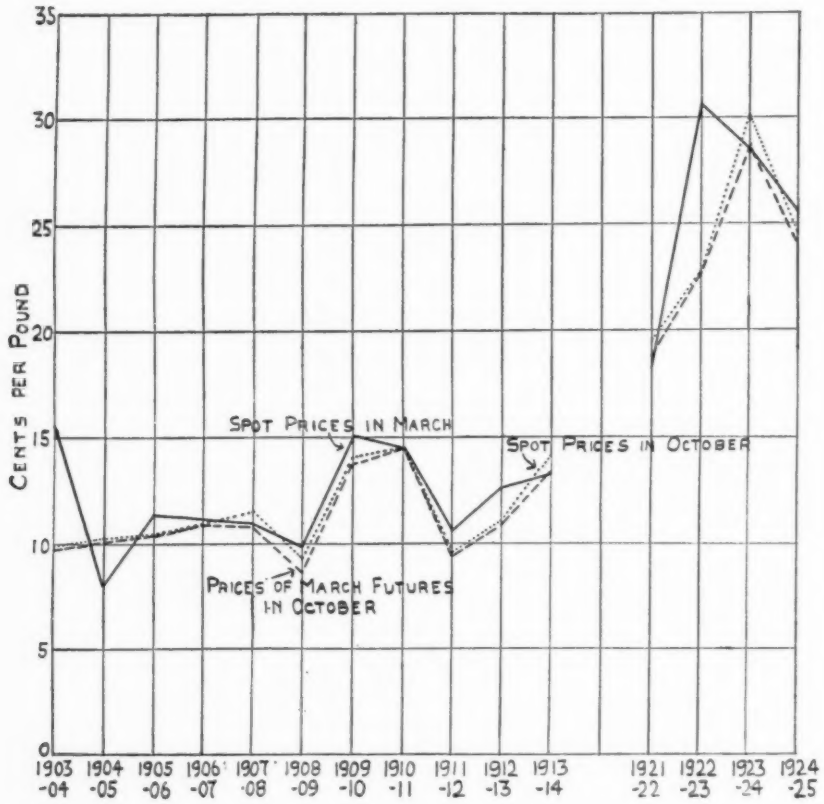
The most commonly accepted forecast of cotton prices is the price of cotton futures. The speculator uses the futures market as a place to pit his judgment of supply, demand, and price movements against the judgments, better or worse, of other speculators. His function is to bring together all the available facts, to act upon them, and thus to

turn the balance of influence toward the maintenance of a fair competitive price. In a sense, therefore, all other advance indicators of cotton prices inhere in the futures price.

However farsighted, shrewd, and industrious, speculative dealers may be in anticipating sizes of crops and of holdings, in attaching just

FIGURE I

COTTON: PRICES OF MARCH FUTURES IN OCTOBER COMPARED WITH SPOT PRICES IN OCTOBER AND WITH SPOT PRICES IN MARCH



the proper shade of meaning to government and private crop reports, and in gathering and piecing together bits of market information from every possible source, they sometimes guess wrong. Speculators' judgments of the future are not free from the dominating influence of the portentous present. Futures prices are consistently more in accord with spot prices of the contracting month than with spot prices of the delivery month. From this one concludes that spot prices have a

strong influence in determining bids and offers for future deliveries, or that the set of immediate causes which make spot prices has a dominating influence in the making of futures prices also.

In Figure I prices of March futures in October are compared with spot prices in October and with spot prices in March. Throughout the period the speculators' guesses in October regarding the price in the following March varied but slightly from the actual October price. The average variation for the years shown was 0.4 cents. When March came, the spot price frequently indicated that the speculators' estimates had been wrong. During the same years the average difference between March spot prices and March prices as forecast by the futures market in October was 2.0 cents. Similar comparisons of May futures in January with spot prices in January and spot prices in May, and of October futures in May with spot prices in May and spot prices in October further demonstrate the same fact. In each case the average futures prices were nearer to the prices in the month the contracts were made than to the prices in the later month which they sought to forecast.

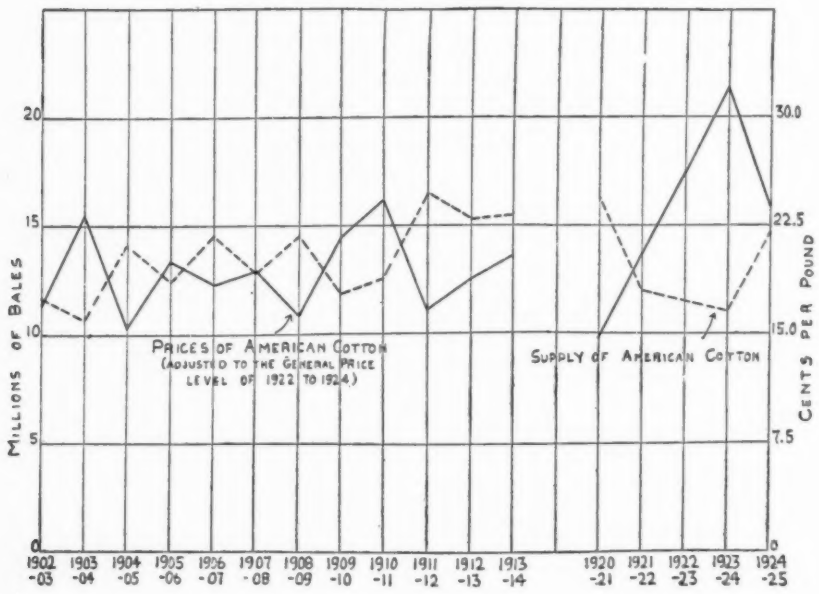
Most students of futures markets agree that new information and new methods of judging old facts have steadily tended to increase the accuracy of price forecasts. However, the above measurements of inaccuracies which still prevail suggest the value of supplementing estimates of the professional cotton trader.

The supply of cotton is an important advance indicator of raw cotton prices. In fact, forecasts based on supply alone have been found to be as accurate as the forecasts of the futures markets although these are presumably based on several factors in addition to supply.

In forecasting cotton prices from the supply of cotton the following method was employed. The figures for supply of cotton used in the computations were something more than annual production. Annual production does not accurately represent the total supply available for consumption because cotton is carried from one crop year to another and the size of the carry-over varies from year to year with conditions of supply and demand. After careful comparisons it was concluded that the most accurate measure of the supply of American cotton was the figure obtained by adding to the United States crop the world carry-over of American cotton. This was more satisfactory than either United States production alone or production in the United States plus carry-over in the United States. It was also found that changes in the world supply of American cotton offered a more adequate explanation of changes in prices of American cotton than did changes in the world supply of all kinds of cotton.

The consistency with which prices respond to changes in supply is illustrated in Figure II. The prices shown in this figure are the annual average spot prices of cotton in terms of a post-war price level—i.e. the actual prices were divided by the Bureau of Labor Statistics average index number of wholesale prices for the crop years 1922–23 and 1923–24. In almost every year an increase in supply of cotton was

FIGURE II
SUPPLY AND PRICE OF COTTON



accompanied by a decrease in price and a decrease in supply by an increase in price. If the supply is to be used for estimating the price it is necessary to have a mathematical measure of the amount of change in price which will be caused by a given change in supply. Such a measure can be obtained by the use of the statistical device of correlation.[1]

When changes in May prices (adjusted to a 1924 price level) over the average prices of April, May, and June for each preceding year were "correlated" with changes in supply over the preceding year, an estimating equation was obtained. This showed that on the average a change of five hundred thousand bales in supply was accompanied

[1] See F. C. Mills, *Statistical Methods*, Chapters on the "Measurement of Relationship" and the "Problem of Estimation"; or other statistical authorities.

by a change of approximately one cent in the May price. With the estimating equation a series of May prices derived from supply were computed. These were equivalent to December estimates of prices for the following May since satisfactory figures for supply are usually available in December. The average difference of the estimated price from the actual price was 1.6 cents, approximately the same in amount as the average difference between May futures prices in December and May spot prices for the same period.

If the same method is applied in estimating in December an average February, March, and April price, the average difference between the actual price and the estimated price is 1.5 cents. The average difference between December prices of March futures and March spot prices is also 1.5 cents. Here again the forecasts and the futures prices fail to coincide with actual spot prices by approximately equal degrees of error.

The accuracy of prices estimated from changes in cotton supplies indicates the important rôle that supply plays in the determination of prices. Changes in supply alone afford a method of forecasting spot prices with as great an accuracy as they are forecasted in the futures markets by futures prices.

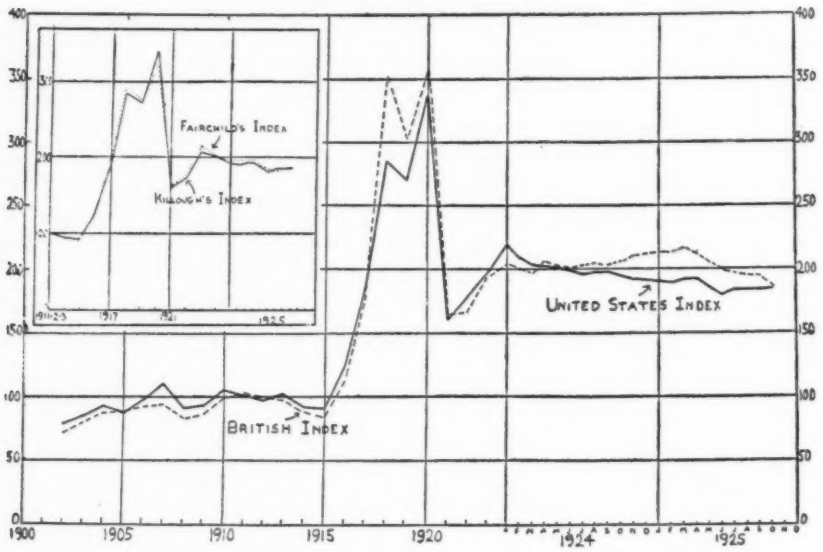
An accurate indicator of the future price of cotton must take into account not only the supply of cotton but also the demand for it. The concept of demand is more difficult to express mathematically than that of supply. Supply can be simply expressed in pounds or bales, but demand is a quantity of two dimensions, including both the amount of cotton purchased and the price paid. To determine demand, therefore, it is necessary to know both the quantity taken and the price.

The major part of the potential demand for raw cotton is embodied in the demand for cotton fabrics. The manufacturers' judgments of future values find expression in terms of cotton cloth prices and volume of production. Cotton cloth prices can be measured by index numbers. Complete statistics of cotton cloth production or of unfilled orders are not available, but statistics of the amounts of cotton consumed in the United States and exported for foreign consumption indicate roughly relative amounts of raw cotton going into the manufacture of cotton fabrics. These data (cotton cloth prices and movements of raw cotton into consumption and export) may be used to estimate raw cotton prices. The cotton cloth merchants avail themselves of futures prices and are in better position than any other group in the cotton trade to anticipate changes in demand for cotton fabrics. Thus the price of cotton fabric reflects the supply of raw cotton as well

as the demand for it, and, therefore, such estimates take both the demand and the supply factors into consideration.

For purposes of this analysis monthly index numbers of cotton goods prices extending back to 1900 were desired. The recognized usefulness of cotton goods index numbers is indicated by the fact that several have come into general use in recent years, but none extend

FIGURE III
INDEX NUMBERS OF COTTON CLOTH PRICES
(Average 1911–1912–1913 = 100)

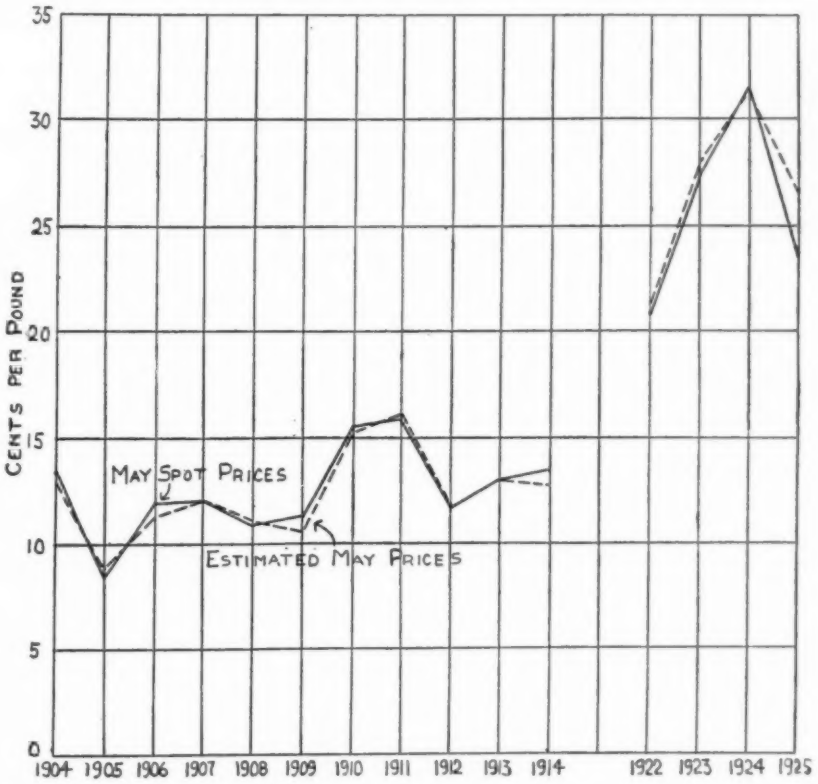


back before 1914. However, prices of nine of the more important items commonly used in such index numbers are available for a quarter of a century or more. These were weighted and computed into an index number beginning in 1900. A similar index number, based on a smaller number of commodities was computed for cotton goods prices in Great Britain. The British and American index numbers are compared in Figure III. The insert in the figure shows the same American index number and the index number published currently in Fairchild's Daily News Record. The Fairchild index number is based on thirty-six cotton fabrics. It is available weekly back to July 8, 1922, monthly to January 1922, and annually as far as 1914. The close comparability of the two index numbers compiled independently shows that they are satisfactory for practical purposes. The British index number compares equally favorably with that published in Frederick

W. Tattersall's *Cotton Trade Review* (Manchester, England). The Tattersall index number goes back to 1914.

A marked divergence between the British and American index numbers occurred in 1917, 1918, and 1919 and again in 1924 and the early months of 1925. The divergence in 1917, 1918, and 1919 was

FIGURE IV
ACTUAL AND ESTIMATED COTTON PRICES



due to such unsettled conditions already mentioned as submarine warfare, lack of shipping facilities, the closing of the New York Cotton Exchange, and government price fixing. The relative increase in cotton cloth prices in Great Britain in 1924 and in 1925 indicated an increasing foreign demand for cotton fabrics, and was accompanied by an increase in the proportion of the American crop exported.

May prices estimated from cotton cloth prices in December and cotton movement from September first to the end of December for fourteen years differed from actual spot prices in May by an average

amount of only 0.4 cents. The average difference for the same years between May futures prices in January and May spot prices was 1.3 cents. The estimated May prices and actual May prices are shown in Figure IV.

Estimates made as of January first for the same years of average spot prices of cotton for the months of February, March, and April, differed from the averages of actual prices for these months by an average amount of 0.7 cents. This error is only one-half as great as the difference between March futures prices in January and March spot prices, i.e. 1.4 cents, or of March futures prices in December and March spot prices (1.5 cents).

The estimates were made by "correlating" raw cotton price changes with changes in the index numbers of cloth prices and changes in the rate of movement of raw cotton into consumption and export trade. For this purpose the movements of raw cotton into consumption and export were converted into index numbers.

Figure IV shows that the estimated price of cotton for May 1925 was above the actual price. This fact directs attention to the influence of spring plantings of cotton upon May prices. The extensive plantings and the favorable growing conditions in the spring of 1925 tended to force spot prices down. The 1925 acreage was the largest ever planted to cotton in this country. Also, the condition of the crop in May was the best since 1918. These factors have not in the past been subject to accurate measurement as early as December. However, valuable results have recently been obtained by estimating plantings as early as December from relative prices prevailing at that time.[1] In the future the inclusion of such estimates in an estimating equation may add to the accuracy of the results presented in Figure IV.

[1] B. B. Smith, "Forecasting the Acreage of Cotton," this JOURNAL, March 1925.

NOTES

COMBINING THE ADVANTAGES OF FIRST-DIFFERENCE AND DEVIATION-FROM-TREND METHODS OF CORRELATING TIME SERIES

By Bradford B. Smith, *United States Bureau of Agricultural Economics*

The statistical investigator is not infrequently required to decide between two methods of correlating time series—the deviation-from-trend and the first-difference. This choice is important, for the two methods often yield correlation coefficients markedly varying in magnitude. This choice of the method is properly made on the basis of the applicability of the implicit assumptions in the two. Certain of these assumptions are in direct contrast.

In the deviation-from-trend method the basic assumption is that there is a linear relation between deviations from trend in one variable and deviations from trend in the other variable. Or, expressing it for convenience in algebraic form,

$$Y - T_y = b_1(X - T_x) \tag{1}$$

wherein Y and T_y represent the dependent variable and associated values of its trend and X and T_x similar values for the independent. Of course, b is the net regression of Y on X.

The (linear) trends of the two series can obviously be stated in terms of the values from which they are computed. Thus

$$\begin{aligned} T_y &= b_2 t + K_2 \\ T_x &= b_3 t + K_3 \end{aligned} \tag{2}$$

wherein t is that numerical designation of time which has been assigned (usually years from middle year), b_2 and b_3 the annual increments (regressions of Y and X, respectively, on t) and K are the appropriate constants (usually the means of Y and X).

If these expressions for trend are substituted in (1) an equation may be formed which shows the type of relationship between the three original measurements, Y, X, and t, which are assumed in correlating deviations from trend. Thus

$$\begin{aligned} Y - b_2 t - K_2 &= b_1(X - b_3 t - K_3) \\ Y &= (K_2 - b_1 K_3) + (b_2 - b_1 b_3)t + b_1 X \end{aligned} \tag{3}$$

Since the terms enclosed in the parentheses are constant, (3) may be written,

$$Y = K + b_4 t + b_1 X \tag{4}$$

which is recognizable as a multiple regression equation in which t and X are the independents. The term, $b_4 t$, may be interpreted as evidencing the trend in the *relationship between* Y and X.[1]

The basic assumption implicit in correlating first differences is that *changes* in Y over its preceding value are a linear function of similar changes in X. Thus

$$Y - Y_{-1} = K + b_1(X - X_{-1}) \tag{5}$$

$$Y = K + Y_{-1} + b_1(X - X_{-1}) \tag{6}$$

A consideration of the "estimating" properties of the two regression equations, (4) and (6), permits throwing into direct contrast the assumptions of the two methods.

If the constants for formulae (4) and (6) be determined by usual methods, the right-hand sides of the equations may be evaluated for associated values of Y, the evaluations being designated Y' or estimates of Y. The difference, $Y - Y'$, is, of course, the residual, Z, or error of estimate.

The implicit assumption of the first-difference method, then, is that whatever influences there were causing an error in estimating for any given observation, these influences tend to persist into the ensuing observation. For, by taking the original value of Y_{-1} as a base in computing an estimate of Y, the error of estimating the original Y_{-1}, or $Y_{-1} - Y'_{-1} = Z_{-1}$, is added to the estimate of that Y_{-1}, or Y'_{-1}, the base thus being changed from one of average or normal relationship, Y'_{-1}, to one which has been corrected for the error, $Y_{-1} = Y'_{-1} + Z_{-1}$, and hence supposing the continuation of influences producing that error.

On the other hand, since a normal or trend value is taken as a base for estimating values in the deviation-from-trend method, the assumption is here implicit that whatever influences there were causing errors of estimate in any given observation, those influences are peculiar to that observation alone and do not persist to the next. The errors are automatically dropped out of the computations by making the estimates for succeeding observations from a trend or normal base.

[1] Formula (4) represents to the author not only an easier arithmetical process for correlating time series than formula (1), but a more appropriate expression of implicit assumptions, for if Y is considered a function of t and X, the correlation coefficient expressing this relationship is $R_{y.tx}$ rather than $r_{xy.t}$. See Bradford B. Smith, "Error in Eliminating Secular Trend and Seasonal Variation Before Correlating Time Series," this JOURNAL, December 1925.

This contrast between the assumptions of the two methods can be easily proved by arbitrarily applying to the first-difference method the assumption of the deviation-from-trend method and showing the identity of the two under these circumstances.

Thus, if in the first-difference method the estimates uncorrected for errors were used as a base for computing succeeding estimates, instead of the estimates corrected by the amount of error, then this assumption could be given algebraic expression by substituting Y'_{-1} for Y_{-1} in (6).

Thus $$Y = K + Y'_{-1} + b_1(X - X_{-1}) \tag{7}$$

But this calls for the value of Y'_{-1} which can only be determined in terms of Y'_{-2} and so on. If carried back to the first case and terms collected, the formula then becomes

$$Y_n = Y'_o + nK + b_1(X_n - X_o) \tag{8}$$

wherein n is a number designating the place in the series of the given observation, and subscript, o, designates the initial value from which the first of the differences in the series was secured. The value of Y'_o is, of course, unobtainable; but it is a constant for all evaluations of the equation, as also is X_o. Hence (8) may be written

$$Y_n = K_2 + nK + b_1 X_n \tag{9}$$

(9) is identical in form with (4), since n varies with t, thus showing the identity of the first-difference and deviation-from-trend methods when the assumptions of the latter are applied to the former. The difference between the two, then, is the difference in using Y' and Y as bases for computing estimates in succeeding years; and this, in turn, is the difference between assuming that the effect of influences causing deviation is peculiar to a given observation in one case, and persists unchanged to the succeeding observation in the other case.

The implicit assumption in the first-difference method makes that method peculiarly suitable for certain types of analyses—notably those concerned with prices which have a large element of mass psychology in them, it being very common and human, for example, to judge a price by comparing it with the preceding year, forgetting whether or not the preceding year was itself out of line. On the other hand, the assumption is of unreasonable rigidity for various other types of analyses. It may be felt that there is some persistence in influences producing errors from year to year; but, on the other hand, it could seem highly improbable that such influences tend to be exerted in the given year to the full and exact force they were exerted in the preceding. Such, however, is the implicit assumption of the first-difference method.

Again, it may seem perfectly feasible that an influence effective in one year may be of a pendulum nature and hence be a reverse influence in the succeeding year. The use of first differences here would aggravate, rather than reduce, the errors of estimate. Evidently what is needed is a method which will permit the inclusion of such persisting influences but allow them importance only in proportion to the degree to which they do persist.

Such a method may be devised as follows:

Equation (6) may be written in the form

$$Y = b_1 Y_{-1} + K + b_2 X + b_3 X_{-1} \tag{10}$$

wherein b_1 must equal 1.0 and b_2 must equal $-b_3$ if the equation is to be identical with equation (6) and, thus, if the assumptions of the first-difference method are strictly appropriate.

But if a solution for the best values of b and K in (10) is made by methods of least squares, the equivalents of b just cited are no longer predetermined by mathematical necessity as in the case of the more usual solution for b and K by methods illustrated in formula (5). Hence these equivalents will only result in case the assumption as to persistence of errors is appropriate. Formula (10) thus typifies a method of correlating time series which has all of the advantages of the first-difference method without that method's rigid implicit assumption. At the same time it obviously has the advantages of the deviation-from-trend method, since X is one of the independents.

Probably in no single instance of correlating time series can it with strict truth be said either (a) that there is no association between influences producing errors in relationship from one observation to the next or (b) that there is a direct, "one-to-one" association. But it is probably equally true that in every instance of correlating time series some element of *both* these hypotheses is present. A correlation of the variables by methods represented in formula (10) permits the data, in themselves, to determine the best values of the constants by criterea of least squares.

If by this method b_1 equals 1.0 and b_2 equals $-b_3$ it is evidence that the assumptions of first-differences are appropriate. To the degree that these fail to result the assumptions are inappropriate. If b_1 and b_3 tend to be very small in comparison with b_2 then the assumptions of deviation-from-trend methods are appropriate. Although experience in using the method shows that it is practically never necessary to introduce the term in t shown in formula (4), such a term may for completeness be introduced in instances where the value and relation-

ship of the b terms indicate appropriateness of deviation-from-trend assumptions. The formula then becomes

$$Y = K + b_1 Y_{-1} + b_2 X + b_3 X_{-1} + b_4 t \tag{11}$$

The constant values may be found by methods of least squares. It is possible to add terms in X_{-2} and Y_{-2} and so on, in case it is felt that errors are of a cyclical nature, or persist over more than one observation. The measure of the relationship of the independents to Y is, of course, given by the coefficient of multiple correlation.

In closing, a more general point of view of this subject may be set forth: When first-differences are correlated the assumption is that any given value of the dependent is a function of three things, i. e.

$$Y = f(Y_{-1},\ X,\ X_{-1}) \tag{12}$$

Methods of solving for the joint function illustrated in (12) are yet in their infancy. The following, however, may be evaluated:[1]

$$Y = f_1(Y_{-1}) + f_2(X) + f_3(X_{-1}) \tag{13}$$

as also can be[2]

$$f_0(Y) = f_1(Y_{-1}) + f_2(X) + f_3(X_{-1}) \tag{14}$$

It is more logical to let the data determine the nature of these functions than arbitrarily to assume them. Correlation method is sufficiently limited by its necessary assumptions as it is.

[1] Mordecai Ezekiel, "A Method of Handling Curvilinear Correlation For Any Number of Variables," this JOURNAL, December, 1924.

[2] Donald Bruce, "On Possible Modifications in the Ezekiel Method of Handling Curvilinear Multiple Correlation," Unpublished Ms. filed in Library of the Department of Agriculture, Washington, D. C.

AN INDEX OF BRITISH BUSINESS CYCLES

By Dorothy Swaine Thomas

In studying the various phenomena of business cycles, it is often desirable to have some composite index of the cycles, from which generalizations as to the *average* state of business at any particular time can be made. Several composites of American business cycles have been made, e. g., those of the Harvard Committee on Economic Research, that of the American Telephone and Telegraph Company, Carl Snyder's index of the volume of trade, and others. These indexes extend over varying periods, and although only approximations to the average fluctuations of trade, they are undoubtedly representative of the general type of fluctuations occurring in many series not included in the composites. For, although research into the business cycle has brought out how infinitely varied are the reactions of different industries, it has also shown the great similarity in the time movement of the larger swings for many diverse types of industry.

No composite index of British business cycles extending back into the nineteenth century has yet (to my knowledge) been published. The Harvard Committee on Economic Research has constructed such an index, by quarters, from 1903 to 1914.[1] The Committee tested many business series for synchronism with the cycles found in the Sauerbeck index of wholesale prices, and accepted the following as showing the general movement of business: exports of British produce, bank clearings in five provincial towns, percentage of unemployment in all trades, and blast furnaces in blast. "The first two register trade—foreign and domestic—while the last two are measures of industrial activity; the former involve a price element, while the latter do not."[2]

Following the lead of this study of British business cycles from 1903 to 1914, I constructed a similar composite index over a much longer period, i. e., by quarters, from 1855 to 1914. My series differ but slightly from those used in the Harvard study. For the period from 1855 to 1881, the only two reliable series available were the value of exports of British produce, and an index of unemployment among ironfounders. From 1881, a series representing freight traffic receipts on sixteen[3] railways was added; from 1885, the Sauerbeck

[1] *Review of Economic Statistics*, June, 1922, Prel. Vol. IV, Supplement 2.

[2] *Op. cit.*, p. 163.

[3] Great Eastern; Great Northern; Great Western; Lancashire and Yorkshire; London, Chatham and Dover; London and Brighton; Manchester, Sheffield and Lincoln; Midland; Northeastern; Southeastern; North London; Caledonian; Glasgow and Southwestern; Great Southern and Western (Ireland); London and Southwestern; North British.

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TABL

Original Items of Serie
(Quarterly aver

A. Value of Total Exports
B. Sauerbeck Index Numbers
C. Blast Furnaces in Blast.
D. Railway Freight Traffic
E. Provincial Bank Clearing
F. Percentage Unemployed I
G. Percentage Unemployed:

	A	B	C	D	E	F	G			A	B	C	
	Million £	£	Number	Million £	Million £	£	£			Million £	£	Number	Mil
1855	6.3					13.1			1875	17.7			
	8.1					10.8				18.9			
	8.7					8.1				19.8			
	8.8					9.5				18.0			
1856	8.4					11.9			1876	17.0			
	9.6					9.0				16.1			
	10.3					7.0				17.3			
	10.3					8.9				16.5			
1857	9.6					9.4			1877	15.8			
	10.7					8.0				16.0			
	11.6					6.6				17.5			
	8.8					13.2				17.1			
1858	7.8					20.6			1878	15.7			
	10.0					19.3				15.9			
	11.0					14.4				16.8			
	10.1					11.5				16.0			
1859	10.2					8.6			1879	14.4			
	10.8					4.5				15.3			
	11.7					3.4				17.1			
	10.8					4.5				17.1			
1860	10.2					3.3			1880	17.4			
	10.5					1.8				18.5			
	13.2					2.5				19.8			
	11.4					4.1				18.7			
1861	9.2					6.5			1881	17.8			2,4
	10.8					8.0				18.7			2,7
	11.2					8.1				20.8			2,6
	10.4					12.3				20.7			3,2
1862	8.8					14.2			1882	19.9			2,8
	10.3					14.2				19.5			2,7
	12.1					12.8				21.3			2,8
	10.1					14.8				19.8			3,0
1863	9.2					14.3			1883	19.7			2,8
	11.5					10.4				19.3			2,8
	14.1					7.8				20.9			2,8
	14.1					5.6				20.0			3,1
1864	12.2					6.6			1884	19.5			2,8
	13.8					4.3				19.1			2,7
	15.1					2.7				20.5			2,7
	12.4					5.2				18.7			3,0
1865	11.9					5.3			1885	17.7	71.2		2,7
	12.8					3.6				17.1	70.4		2,7
	15.2					2.8				18.8	70.0		2,7
	15.4					3.3				17.5	69.4		2,9
1866	15.7					4.9			1886	17.4	68.2		2,7
	15.3					5.1				17.1	65.9		2,6
	16.4					6.4				18.7	66.8		2,6
	15.6					10.2				17.6	67.7		2,9
1867	14.1					13.7			1887	18.0	67.3		2,7
	15.1					15.4				16.7	65.8		2,7
	16.5					14.6				19.6	68.7		2,6
	14.7					19.3				19.4	68.9		3,2
1868	13.8					20.9			1888	18.9	70.7		2,8
	14.4					17.0				18.8	66.9		2,8
	16.4					14.6				20.6	68.1		2,8
	15.2					15.7				19.9	71.4		3,1
1869	14.3					17.1			1889	20.2	70.9		3,0
	16.3					14.8				19.6	68.4		2,9
	17.0					12.7				21.1	69.9		3,0
	15.8					14.6				22.1	73.2		3,3
1870	15.2					11.4			1890	20.9	72.0		3,1
	17.1					5.6				21.6	70.3		3,0
	17.2					4.4				23.3	71.1		3,1
	17.0					6.3				22.0	70.4		3,3
1871	15.5					5.5			1891	20.7	69.2		3,1
	18.4					2.0				20.7	68.5		3,2
	21.3					1.2				21.1	67.7		3,1
	19.2					1.6				19.9	67.1		3,4
1872	19.2					1.0			1892	19.4	66.0		2,9
	20.7					0.9				17.9	65.2		3,2
	23.6					1.1				19.5	63.2		3,2
	22.0					2.8				18.9	64.9		3,7
1873	20.8					2.9			1893	18.2	66.2		3,1
	21.1					2.2				17.8	63.8		3,0
	22.6					4.1				19.2	64.5		2,8
	20.5					3.9				17.6	66.0		3,2
1874	19.3					4.2			1894	18.0	62.7		3,2
	20.0					4.1				17.7	61.1		3,2
	21.0					3.6				18.2	60.3		3,1
	19.5					4.2				18.1	58.5		3,4

TABLE X

ters of Series of British Business Statistics -
uarterly averages of monthly items)

tal Exports of British Produce.
ndex Numbers of "All Materials".
ces in Blast.
ight Traffic Receipts (selected railways).
Bank Clearings (Manchester and Birmingham).
Unemployed Ironfounders.
Unemployed: "All Trades".

C	D	E	F	G		A	B	C	D	E	F	G
	Million	Million				Million			Million	Million		
Number	£	£	%	%		£	%	Number	£	£	%	%
			3.8		1895	17.6	57.3		3.14	15.4		7.5
			2.8			17.8	59.6		3.15	14.9		6.0
			3.4			20.2	61.9		3.25	16.8		5.1
			4.4			19.8	62.4		3.60	17.5		4.7
			4.9		1896	20.4	61.4		3.38	18.5		3.9
			5.1			19.3	59.9		3.30	18.3		3.2
			6.0			20.5	59.4		3.38	19.1		3.4
			7.6			19.8	60.9		3.75	19.2		3.1
			7.4		1897	19.8	60.2	350	3.48	20.9		2.9
			7.6			19.4	59.4	350	3.41	18.7		2.5
			8.3			19.5	60.1	345	3.49	18.3		3.5
			13.2			19.4	59.3	350	3.80	18.6		4.9
			13.9		1898	19.2	59.9	354	3.69	20.3		3.9
			14.4			18.3	62.1	344	3.56	19.0		2.5
			13.4			20.1	62.0	347	3.59	19.2		2.4
			16.9			20.2	62.5	363	3.97	21.1		2.3
			23.9		1899	20.7	65.3	376	3.83	21.5	/	2.3
			23.7			21.5	67.1	380	3.85	20.5		1.8
			22.2			22.6	70.9	380	3.77	20.8		1.9
			18.6			23.4	76.8	380	4.08	22.4		2.0
			13.1		1900	24.0	81.3	381	3.98	24.1		2.2
			9.9			24.1	79.6	382	3.94	22.9		2.0
			9.9			24.7	79.8	369	3.93	22.2		2.6
			9.5			24.2	77.4	337	4.09	23.6		3.0
	2.49		10.8		1901	23.6	73.7	297	4.00	23.7		3.3
	2.70		8.4			22.6	72.1	302	3.84	21.2		3.1
	2.60		6.0			23.5	71.7	318	3.87	21.7		3.2
	3.28		5.7			23.6	71.0	321	4.22	21.9		3.6
	2.82		5.3		1902	22.6	71.3	320	4.02	23.6		3.7
	2.73		4.0			22.5	71.7	322	3.98	21.8		3.5
	2.80		3.4			24.7	71.0	326	3.98	21.7		4.0
	3.05		4.8			24.6	70.8	322	4.41	22.6		4.6
	2.85		4.5		1903	24.3	72.4	329	4.08	23.1		4.4
	2.88		4.0			23.2	71.8	325	4.03	21.8		3.7
	2.88		4.2			25.0	71.2	319	4.03	21.9		4.9
	3.12		4.9			24.5	72.1	310	4.30	22.5		5.8
	2.83		6.1		1904	24.1	74.2	299	4.12	23.9		5.7
	2.79		6.6			24.0	71.4	306	4.07	22.2		5.6
	2.77		7.4			25.7	71.3	301	4.00	22.3		5.9
	3.02		9.5			26.5	72.5	303	4.36	24.3		6.6
	2.78		10.4		1905	26.1	72.3	311	4.19	24.9		5.7
	2.72		9.7			25.8	73.0	321	4.03	23.6		4.9
	2.70		10.8			28.9	75.2	323	4.11	24.1		4.8
	2.95		12.6			29.2	78.5	333	4.50	26.2		4.5
	2.71		14.8		1906	30.4	80.1	337	4.38	27.1		3.9
	2.67		14.0			29.8	82.5	340	4.26	25.8		3.2
	2.67		12.8			32.5	83.0	339	4.24	26.3		3.2
	2.92		13.2			32.5	86.9	342	4.59	28.9		4.1
	2.74	12.3	11.4	8.8	1907	34.0	88.2	345	4.52	30.1		3.5
	2.72	11.0	9.6	7.7		34.8	88.7	347	4.50	28.4		3.0
	2.69	11.8	9.2	8.1		37.7	85.1	345	4.53	28.8		3.6
	3.24	12.5	9.3	8.0		35.6	80.1	323	4.79	30.1		4.8
	2.85	12.8	7.2	6.8	1908	33.1	76.0	299	4.49	29.9		6.1
	2.83	12.2	5.8	4.9		30.3	73.2	287	4.51	26.1		6.6
	2.85	12.7	5.3	4.4		31.9	73.1	271	4.53	25.6		8.6
	3.10	13.2	3.8	3.6		30.5	74.0	289	4.55	24.5		9.1
	3.01	14.1	2.9	2.7	1909	29.6	73.0	285	4.57	26.9		8.4
	2.95	13.9	1.8	1.9		29.4	74.1	287	4.59	25.0		8.0
	3.00	13.2	1.7	2.1		33.5	75.9	292	4.60	25.7		7.7
	3.37	14.9	1.4	1.7		33.7	78.2	300	4.62	27.4		6.7
	3.13	15.8	1.9	1.5	1910	33.6	80.0	303	4.64	27.7		5.9
	3.07	15.5	2.8	1.9		34.6	79.9	311	4.65	28.3		4.1
	3.15	16.0	2.6	2.4		38.0	80.4	311	4.67	27.3		4.0
	3.39	16.0	2.7	2.7		37.3	82.7	308	4.69	30.3		4.7
	3.37	16.7	3.9	2.9	1911	38.1	83.6	309	4.71	32.2		3.4
	3.20	15.4	4.3	2.5		36.5	84.5	292	4.72	29.8		2.8
	3.17	15.7	4.9	4.0		35.8	81.9	285	4.74	29.5		3.0
	3.46	15.9	5.9	4.2		41.0	81.9	291	4.76	32.1		2.8
	2.94	16.6	7.4	5.5	1912	39.5	84.2	207	4.77	31.2		5.6
	3.26	14.8	8.0	5.5		35.6	86.7	262	4.79	29.7		2.9
	3.20	14.6	8.2	5.4		43.0	89.7	325	4.80	32.3		2.3
	3.71	15.8	11.4	8.6		44.4	91.7	328	4.82	34.2		2.0
	3.11	15.0	10.9	9.4	1913	42.4	91.6	334	4.83	34.6		2.0
	3.01	14.3	8.5	6.3		43.3	90.9	332	4.84	34.2		1.8
	2.82	15.3	9.7	6.9		44.6	91.1	316	4.86	33.4		2.1
	3.25	15.7	14.6	7.5		44.9	90.1	288	4.87	34.8		2.3
	3.26	16.3		6.6	1914	44.5	89.3	275	4.89	35.2		2.3
	3.26	15.3		6.2		40.6	86.9	267	4.90	32.4		2.3
	3.10	15.4		7.6								
	3.40	15.8		7.4								

TABLE II

British Business Cycl
Percentage Deviations of Acual Items (wit
for) from Secular Trend, in terms of Stan

	A	B	C	D	E	F	G	AVERAGE
1855	-1.66					- .45		-1.06
	- .26					- .33		- .30
	- .33					+.03		- .15
	+ .33					+ .06		+ .20
1856	+ .38					- .25		+ .07
	+1.04					- .02		+ .51
	+ .86					+ .23		+ .55
	+1.23					+ .17		+ .70
1857	+1.11					+ .18		+ .65
	+1.43					+ .15		+ .79
	+1.46					+ .28		+ .87
	-1.18					- .62		- .90
1858	-1.55					-1.82		-1.69
	+ .07					-1.88		- .91
	+ .18					-1.12		- .47
	- .28					- .33		- .31
1859	+ .55					+ .30		+ .43
	+ .26					+ .76		+ .51
	+ .11					+ .83		+ .47
	- .17					+ .92		+ .38
1860	- .02					+1.24		+ .61
	- .63					+1.25		+ .31
	+1.04					+1.01		+1.03
	- .17					+ .99		+ .41
1861	-1.48					+ .67		- .41
	- .85					+ .11		- .37
	-1.42					- .02		- .72
	-1.43					- .51		- .97
1862	-2.27					- .75		-1.51
	-1.62					-1.03		-1.33
	-1.16					- .89		-1.03
	-2.22					- .99		-1.61
1863	-2.30					- .79		-1.55
	-1.13					- .35		- .74
	+ .15					+ .02		+ .09
	+ .66					+ .70		+ .68
1864	- .16					+ .63		+ .24
	+ .38					+ .77		+ .58
	+ .40					+ .95		+ .68
	-1.19					+ .77		- .21
1865	- .80					+ .86		+ .03
	- .87					+ .89		+ .01
	+ .97					+ .93		+ .50
	+ .77					+1.12		+ .95
1866	+1.73					+ .93		+1.33
	+ .87					+ .61		+ .64
	+ .49					+ .25		+ .37
	+ .50					- .19		+ .16
1867	+ .10					- .74		- .32
	+ .07					-1.35		- .64
	+ .08					-1.31		- .62
	- .46					-1.93		-1.20
1868	- .43					-2.13		-1.28
	- .76					-1.67		-1.22
	- .38					-1.32		- .85
	- .55					-1.27		- .91
1869	- .39					-1.42		- .91
	+ .22					-1.27		- .53
	- .34					- .99		- .67
	- .46					-1.08		- .76
1870	- .08					- .35		- .22
	+ .39					+ .49		+ .44
	- .46					+ .61		+ .08
	+ .06					+ .52		+ .29
1871	- .15					+ .78		+ .32
	+ .95					+1.18		+1.07
	+1.76					+1.23		+1.50
	+1.11					+1.42		+1.27
1872	+1.89					+1.65		+1.77
	+2.03					+1.39		+1.71
	+2.90					+1.24		+2.07
1873	+2.54					+1.19		+1.87
	+1.48					+1.28		+1.38
	+2.06					+1.14		+1.66
	+1.50					+ .65		+1.08
1874	+1.44					+..96		+1.20
	+1.47					+1.02		+1.25
	+1.15					+ .76		+ .96
	+ .81					+ .74		+ .78
	+ .63					+ .90		+ .77

	A	B	C	D	E
1875	+ .36				
	+ .38				
	- .02				
	- .26				
1876	- .17				
	-1.39				
	-1.62				
	-1.41				
1877	- .96				
	-1.54				
	-1.60				
	-1.15				
1878	-1.08				
	-1.73				
	-2.06				
	-1.80				
1879	-1.90				
	-2.07				
	-1.92				
	-1.26				
1880	- .26				
	- .33				
	- .45				
	- .36				
1881	- .04			-2.87	
	- .27			- .56	
	+ .11			-2.16	
	+ .79			+2.76	
1882	+ .58			+ .18	
	+ .58			+ .20	
	+ .98			+ .43	
	+ .40			+3.16	
1883	+ .63			+ .46	
	+ .67			+1.17	
	+ .87			+1.20	
	+ .56			+1.85	
1884	+ .57			+ .31	
	+ .55			+ .31	
	+ .65			+ .10	
	+ .17			+ .15	
1885	- .49	- .23		- .16	
	- .58	- .31		- .56	
	- .34	- .25		- .77	
	- .88	- .31		+ .33	
1886	- .65	- .47		-1.20	
	- .57	- .87		-1.85	
	- .39	- .65		-1.22	
	- .81	- .33		-1.17	
1887	- .31	- .33		-1.22	- .02
	- .80	- .58		-1.07	- .93
	+ .12	- .54		-1.40	- .12
	+ .23	+ .23		+1.30	- .08
1888	+ .15	+ .76		- .66	- .42
	+ .35	- .09		- .43	- .32
	+ .64	+ .31		- .33	+ .05
	+ .44	+1.11		- .36	- .20
1889	+ .90	+1.07		+ .46	+ .30
	+ .68	+ .59		+ .23	+ .95
	+ .95	+ .97		+ .54	- .15
	+1.71	+1.82		+1.53	+1.00
1890	+1.29	+1.60		+ .94	+1.29
	+1.95	+1.22		+ .76	+1.96
	+2.12	+1.44		+1.30	+2.30
	+1.60	+1.28		+1.10	+1.39
1891	+1.12	+1.06		+ .76	+1.45
	+1.37	+ .90		+1.25	+ .91
	+ .30	+ .76		+ .94	+1.11
	+ .33	+ .63		+1.10	+ .47
1892	+ .31	+ .39		-1.58	+..45
	- .36	+ .23		+1.20	- .42
	- .15	- .25		+ .64	- .81
	- .25	+ .14		+2.58	- .43
1893	- .45	+ .47		- .66	-1.83
	- .44	- .09		-1.22	-1.58
	- .45	+ .05		-2.81	- .66
	-1.14	+ .44		-1.68	-1.11
1894	- .66	- .37		- .18	-1.16
	- .63	- .78		+ .18	-1.13
	-1.10	-1.01		-1.20	-1.21
	-1.01	-1.47		-1.17	-1.63

E II

siness Cycles
Items (with seasonal variation allowed
rms of Standard Deviation.

D	E	F	G	AVERAGE		A	B	C	D	E	F	G	AVERAGE
		+1.09		+ .73	1895	-1.02	-1.79		-1.53	-2.61		-1.31	-1.65
		+1.01		+ .70		- .69	-1.22		-1.20	-2.09		-1.09	-1.26
		+ .78		+ .38		- .11	- .68		- .59	- .48		- .27	- .43
		+ .69		+ .22		- .18	- .58		- .18	- .65		+ .06	- .31
		+ .86		+ .35	1896	+ .36	- .86		= .33	- .33		+ .42	- .15
		+ .54		- .33		- .04	-1.27		- .64	+ .45		+ .47	- .21
		+ .24		- .69		- .20	-1.40		- .24	+1.10		+ .53	- .04
		+ .20		- .61		- .39	-1.07		+ .20	+ .35		+ .83	- .02
		+ .35		- .31	1897	- .17	-1.26	+ .35	- .20	+1.15		+ .89	+ .13
		+ .03		- .76		- .20	-1.49	- .46	- .46	+ .20		+ .80	- .27
		- .23		- .92		- .92	-1.34	- .63	- .02	- .25		+ .47	- .45
		- .95		-1.05		- .88	-1.59	-..37	- .02	- .81		- .07	- .62
		-1.03		-1.03	1898	- .84	-1.48	- .13	+ .71	+ .07		+ .35	- .22
		-1.37		-1.55		-1.00	- .98	- .56	+ .02	- .05		+ .79	- .30
		-1.28		-1.67		- .85	-1.09	- .38	+ .05	- .05		+1.01	- .22
		-1.72		-1.76		- .70	-1.03	+ .48	+ .46	+ .70		+1.20	+ .18
		-3.05		-2.48	1899	- .16	- .41	+1.10	+1.02	+ .46		+1.17	+ .53
		-3.31		-2.69		+ .33	- .03	+1.32	+1.40	+ .60		+1.13	+ .79
		-3.12		-2.52		+ .08	+ .75	+1.41	+ .82	+ .70		+1.25	+ .84
		-2.12		-1.69		+ .55	+2.05	+1.43	+ .59	+1.16		+1.35	+1.19
		- .86		- .56	1900	+1.03	+3.07	+1.53	+1.43	+1.96		+1.21	+1.71
		- .48		- .41		+1.22	+2.62	+1.61	+1.40	+1.96		+1.03	+1.64
		- .48		- .52		+ .71	+2.52	+1.12	+1.25	+1.25		+ .89	+1.29
		- .59		- .30		+ .61	+1.91	- .27	+ .08	+1.51		+ .84	+ .75
		- .24		-1.10	1901	+ .48	+1.01	- .48	+ .99	+ .91		+ .68	+ .59
		- .39		- .18		+ .13	+ .61	- .22	+ .15	+ .08		+ .47	+ .20
		- .18		- .34		- .24	+ .45	-1.02	+ .23	+ .30		+ .58	+ .05
		- .62		+1.36		- .12	+ .17	- .82	+ .28	- .28		+ .53	- .04
		+ .54		+ .50	1902	- .39	+ .20	- .78	+ .51	+ .37		+ .44	+ .06
		+ .74		+ .50		- .30	+ .17	- .60	+ .48	.00		+ .25	.00
		+ .73		+ .72		- .14	- .03	- .32	+ .38	- .22		+ .17	- .03
		+ .74		+1.43		- .08	- .12	- .48	+ .89	- .37		+ .03	- .02
		+ .72		+ .66	1903	- .07	+ .16	= .08	+ .33	- .61		+ .07	- .02
		+ .90		+ .85		- .41	- .08	- .19	+ .25	- .58		+ .15	- .14
		+ .72		+ .88		- .48	- .25	- .38	+ .15	- .61		- .31	- .31
		+ .57		+ .83		- .60	- .14	- .78	- .46	- .96		- .63	- .60
		+ .69		+ .48	1904	- .57	+ .27	-1.28	- .08	- .63		- .60	- .48
		+..55		+ .34		- .48	- .45	- .83	- .10	- .81		- .83	- .58
		+ .16		+ .21		-..22	- .56	-1.05	- .54	- .86		- .84	- .68
		- .12		- .30		- .21	- .37	- .87	- .59	- .27		-1.05	- .56
		- .30		- .07	1905	- .63	- .47	- .38	- .13	- .50		- .62	- .46
		- .38		- .37		- .20	- .42	+ .16	- .79	- .35		- .50	- .35
		- .52		- .49		+ .16	- .03	+ .32	- .43	- .13		- .29	- .07
		- .87		- .56		+ .34	+ .59	+ .85	- .33	+ .47		+ .01	+ .32
		- .99		- .46	1906	+ .96	+ .81	+1.11	+ .48	+ .43		+ .29	+ .68
		-1.36		- .92		+ .82	+1.23	+1.37	+ .10	+ .60		+ .36	+ .75
		-1.47		-1.04		+ .96	+1.24	+1.37	- .15	+ .78		+ .53	+ .79
		-1.32		- .90	1907	+1.05	+1.90	+1.59	- .31	+1.63		+ .21	+1.01
		-1.14		- .86		+1.68	+2.11	+1.77	+ .79	+ .97		+ .48	+1.15
- .02		- .80	-1.74	- .74		+2.03	+2.15	+1.94	+ .94	+1.55		+ .45	+1.53
- .93		- .53	-1.52	- .91		+2.16	+1.35	+1.89	+1.05	+1.71		+ .31	+1.41
- .12		- .56	-1.54	- .67	1908	+1.48	+ .28	+ .81	+ .36	+1.75		- .18	+ .75
- .08		- .30	-1.35	+ .01		+ .69	- .64	- .38	+ .43	+ .88		- .89	+ .62
- .42		+ .27	- .84	- .11		- .19	-1.28	- .95	- .56	- .38		-1.45	- .80
- .32		+ .30	- .25	- .07		- .50	-1.40	-1.75	- .33	- .84		-2.33	-1.19
+ .05		+ .29	+ .15	+ .18		- .88	-1.24	- .76	-2.14	-2.28		-2.45	-1.63
- .20		+ .90	+ .67	+ .43	1909	-1.06	-1.49	- .98	- .66	-1.46		-2.12	-1.28
+ .30		+1.21	+1.05	+ .83		-1.03	-1.34	- .79	- .89	-1.61		-2.21	-1.31
+ .95		+1.17	+1.15	+ .79		- .58	-1.12	- .24	- .43	-1.30		-1.88	-1.93
- .15		+1.08	+1.22	+ .77		- .47	- .71	- .06	-1.38	- .95		-1.22	- .80
+1.00		+1.42	+1.55	+1.51	1910	- .37	- .50	+ .16	- .43	-1.51		- .83	- .58
+1.29		+1.42	+1.60	+1.36		+ .03	- .61	+ .60	+ .18	- .23		- .16	+ .03
+1.96		+ .95	+1.14	+1.33		+ .17	- .56	+ .60	+ .02	- .90		+ .05	+ .10
+2.30		+ .88	+1.06	+1.52		+ .02	- .19	+ .45	- .71	- .03		- .17	- .11
+1.39		+1.13	+1.07	+1.26	1911	+ .36	- .09	+ .48	+ .69	+ .33		+ .49	+ .38
+1.45		+ .97	+ .94	+1.05		.00	- .08	- .41	- .80	+ .03		+ .52	- .02
+ .91		+ .60	+ .85	+ .98		-1.09	- .61	- .79	- .38	- .25		+ .68	- .42
+1.11		+ .35	+ .30	+ .71	1912	+ .39	- .68	- .43	+ .43	+ .33		+ .83	+ .15
+ .47		+ .40	+ .35	+ .55		+ .09	- .36	•	•	- .81		- .71	- .45
+..45		+ .17	- .30	- .09		- .89	+ .11	•	- .38	- .60		+ .46	- .26
- .42		- .25	- .59	- .03		+ .18	+ .50	+1.28	+1.92	+ .61		+ .95	+ .91
- .81		- .40	- .38	- .23		+ .55	+ .81	+1.43	+1.05	+ .78		+1.22	+ .98
- .43		- .86	-1.73	- .11	1913	+ .17	+ .73	+1.75	+ .24	+ .30		+1.20	+ .74
-1.83		- .65	-2.34	- .91		+ .44	+ .48	+1.59	+1.15	+1.10		+1.05	+ .97
-1.58		- .38	- .98	- .78		- .11	+ .48	+ .73	+ .24	+ .55		+1.05	+ .49
- .66		- .77	-1.11	- .96		.00	+ .25	- .98	- .08	+ .42		+1.09	+ .12
-1.11		- .93	-1.25	- .95	1914	+ .02	- .16	-1.64	+ .59	+ .47		+1.05	+ .06
-1.16			- .86	- .65		- .82	- .65	-2.12	- .46	- .42		+ .77	- .62
-1.13			- .94	- .66									
-1.21			-1.46	-1.20									
-1.63			-1.22	-1.30									

• Omitted because of Coal Strike.
In columns F and G, the signs are reversed.

index of materials, and from 1887, bank clearings in two[1] large provincial towns and the percentage unemployed in all trades were added; except that the percentage of unemployed ironfounders was dropped out beginning with 1894. Finally, from 1897, the series representing blast furnaces in blast, was included.[2]

TABLE III

(1) Formulae for Lines of Secular Trend for Series of British Business Statistics

Series	Period	Origin	Formula
A	1854–1884	1869	$y = 16.22 + .473x - .01358x^2 - .0004881x^3$
	1885–1913	1899	$y = 21.73 + .638x + .04799x^2 + .001225x^3$
B	1885–1913	1899	$y = 67.45 + .978x + .0636x^2 - .00196x^3$
C	1897–1913	1905	$y = 317.41 - 5.09x + .2105x^2 + .02394x^3$
D	1881–1913	1897	$y = 3.572 + .085x + .001046x^2 - .000085x^3$
E	1887–1913	1900	$y = 21.1 + .701x + .00915x^2 + .00006207x^3$
F	1855–1893	1874	$y = 8.3282 - .05808x$
G	1860–1912	1886	$y = 4.489 - .017x$

(2) Indices of Seasonal Variation for Series of British Business Statistics

(By quarters)

Series	A	A	D	E	F	G
Period	1854–1881	1882–1914	1881–1914	1887–1914	1855–1893	1887–1914
First Quarter	93	99	99	104	112	104
Second Quarter	99	97	98	97	95	91
Third Quarter	107	103	97	98	88	99
Fourth Quarter	101	101	106	101	105	106

There is no marked seasonal variation in Series B and C.

(3) Standard Deviations of Percentage Deviations of Actual Items from Secular Trend for Series of British Business Statistics

Series	Period	Standard Deviation
A	1854–1914	9.25%
B	1885–1914	6.44%
C	1895–1914	6.30%
D	1881–1914	3.93%
E	1887–1914	6.03%
F	1855–1893	60.5%
G	1887–1914	45.2%

[1] Manchester and Birmingham.

[2] The sources of the data are as follows:

A. Board of Trade, *Monthly Statement of Trade and Navigation.*

B. 1885–1911, March issues of the *Journal of the Royal Statistical Association.*
1912–1914, *The Statist.*

C. *The Labour Gazette.*

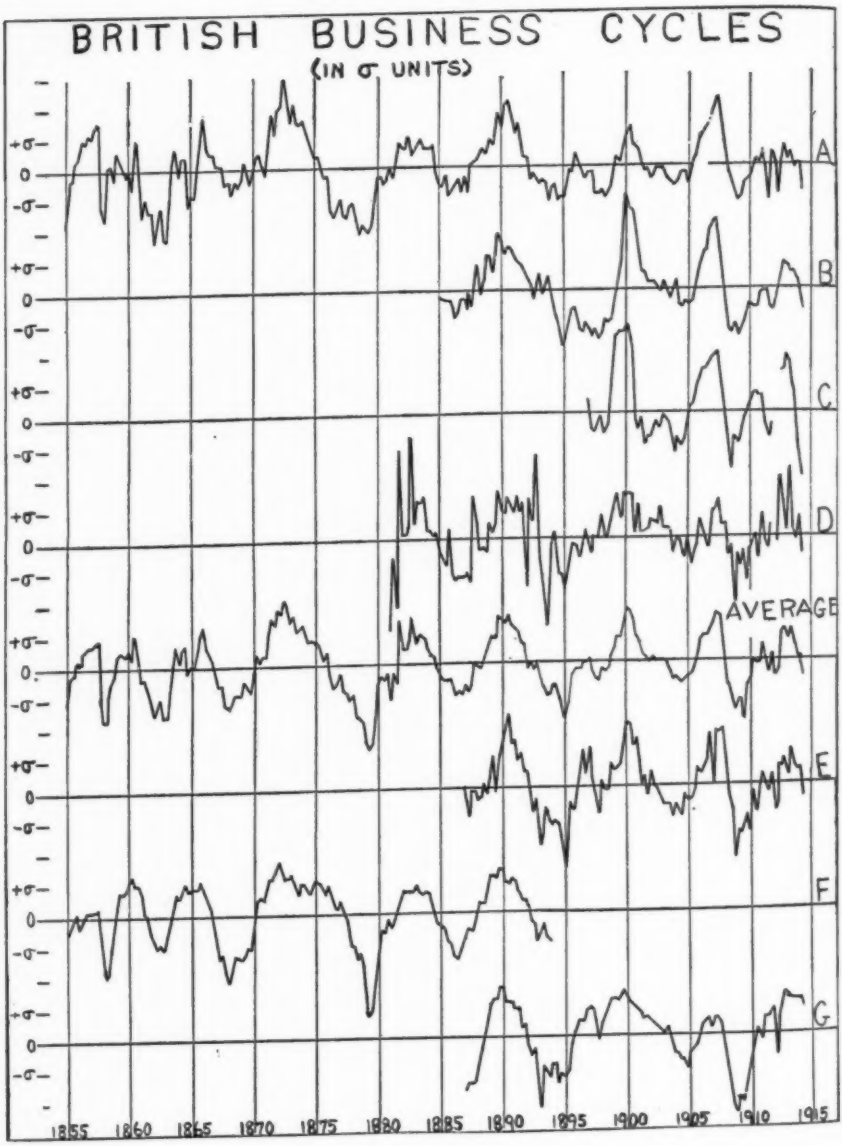
D. *The Statist.*

E. *The Banker's Magazine* (London).

F. Bowley, *Elements of Statistics* (1920 edition), p. 161.

G. *Abstract of Labour Statistics* (and *Labour Gazette*).

(The letters refer to columns in Tables I and III.)


BRITISH BUSINESS CYCLES
(IN σ UNITS)
+σ
0
-σ
A
B
C
D
AVERAGE
E
F
G
1855
1860
1865
1870
1875
1880
1885
1890
1895
1900
1905
1910
1915

The statistical analysis of the series was made, in general, in the Harvard tradition, i. e., by measuring the fluctuations of each series (corrected for seasonal variation) as percentage deviations of the original items from the line of secular trend, and expressing these deviations in terms of their standard deviation. In fitting the secular trends, however, I used parabolae rather than the less flexible straight lines. Seasonal variation was measured by the Harvard "link-relative" method.

The composite index is a simple average of the series listed. As the number of series used in the average increases, the individual peculiarities of the individual series tend to be ironed out, and the composite becomes more representative of the cyclical movement in business generally. No attempt was made to weight the components of the average. The data are presented, in the accompanying tables, in full, in the hope that other investigators may be able to utilize them, perhaps in forming a weighted composite, or in adding other comparable series to my composite, or in using the individual series for various purposes.

The chart shows the striking synchronism of the major movements in the several series, and the composite curve shows how well erratic movements peculiar to each series were ironed out in the average.

While never losing sight of the fact that the best composite index must necessarily cover up many important manifestations of the business cycle, there are times when the investigator needs just such a summary series. I have found this index valuable as an economic standard with which to compare fluctuations in various social phenomena—marriages, births, deaths, crime, etc.,—and doubtless its use in this way could be extended to other fields.

INTERNATIONAL STATISTICAL INSTITUTE

SIXTEENTH SESSION

The International Statistical Institute held its sixteenth biennial session at Rome September 26—October 2, 1925. At the formal opening meeting it was announced that the business sessions would be organized in three sections meeting simultaneously. The first section under the presidency of Professor Benini dealt with questions of statistical method and of demography; the second under the presidency of M. Colson with questions of economic statistics; the third under the presidency of M. Sauveur with questions of social statistics.

The first section discussed the classification of industries based on a report by March, the representative method based on a report by Jensen, methods of certifying the fact and the cause of death based on a report by Huber, and the statistics of international migrations based on a paper by Willcox.

The second section discussed the census of manufactures based on a report by Flux, the statistics of stocks of raw materials based on a report by Methorst, the year books of great cities based on a report by Thirring, and the international statistics of production in fields open to monopoly or subject to an excise tax based on a report by Julin.

The third section discussed the statistics of the cost of living based on a report by Giusti, the statistics of industrial accidents based on a paper by Ney, the statistics of unemployment based on a report by Hilton, and the statistics of wages and hours of labor based on a paper by Huber.

Most of these reports emanated from committees, the appointment of which had been ordered at the session of the Institute held at Brussels in 1923 and made later by the Executive Committee. In addition a number of communications were received by the Institute. Two of these were contributed by Americans, one by Professor W. F. Ogburn, of Columbia University, upon the relationship between marital condition and death, crime, insanity, and pauperism; the other by L. M. Estabrook, American delegate to the International Institute of Agriculture, on the plan for a world census of agriculture in 1930.

The session was more fully attended than the previous one at Brussels in 1923 and was more thoroughly international. Many German and Austrian members were present and they and their language were welcomed on terms of full equality which would have hardly been possible in 1923, or at Brussels. The change which had been wrought in two years was marked and gratifying.

The Institute was entertained with gracious and lavish hospitality. The meetings were in the beautiful palace given some years ago by the King of Italy to the International Institute of Agriculture. Receptions, teas, dinners, and excursions were numerous and delightful. Among the last were a visit by special train to the excavations at old Ostia ending with a tea a few miles away at the modern sea side resort of the same name, an automobile drive along the Appian Way and to several of the loveliest spots in the Alban Hills, and to crown the end of the scientific sessions a two days' trip to Naples and Pompeii. The Italian Committee of Arrangements provided thus a social program which could not elsewhere be equalled for the world has only one Italy, only one Rome.

For the meeting in 1927 the arguments and inducements of the young government of Egypt prevailed and the General Assembly finally voted that the seventeenth session should be held in Cairo.

WALTER F. WILLCOX

THE CHOICE OF A CLASS INTERVAL

CASE I. COMPUTATIONS INVOLVING A SINGLE SERIES

In case a single statistical series of range R with N items is involved in a computation, the optimal class interval may be estimated from the formula

$$C = \frac{R}{1 + 3.322 \log N}$$

This formula gives the class interval for the computation of the averages, measures of dispersion, skewness, etc., of frequency distributions. It is based on the principle that the proper distribution into classes is given, for all numbers which are powers of 2, by a series of binomial coefficients. For example, 16 items would be divided normally into 5 classes, with class frequencies 1, 4, 6, 4, 1. Thus if a statistical series had 16 items with values ranging from 20 to 70, or a range of 50 points, it should be divided into 5 classes of 10 points each, that is, the class interval would be 10. Similarly, 64 is the sixth power of 2, so a statistical series containing 64 items should be divided into 6 plus 1, or 7 classes. If such a series had a range of 35 points the class interval would be 5.

The most convenient class intervals are 1, 2, 5, 10, 20, etc., so that in practice the formula for the theoretical class interval may be used as a means of choosing among these convenient ones. In general the next smaller convenient class interval should be chosen, that is, the one next below the theoretically optimal interval. If the formula gives 9, 10 may be chosen, but if the formula indicates 7 or 8, the one actually used should generally be the next lower convenient class interval, 5.

CASE II. COMPUTATIONS INVOLVING TWO SERIES

In computing the coefficient of correlation, if the approximate value, r, of the index is known in advance, the optimal class interval to be used in each series may be estimated from the formula

$$C = \frac{R}{1 + (1.661 + 1.661r) \log N}$$

For example, in many biological correlations the approximate value of r is known to be 0.50, which results in 2.491, or approximately 2.5 as the coefficient of $\log N$ in the formula.

In case there is no advance knowledge of the value of r it should be first assumed to be 0, which gives the largest class intervals and the smallest number of classes, in other words the easiest computation. If the value of r derived from the preliminary computation is much different from 0, it may be used in a more accurate second determination of the class intervals in the series concerned, and r may then be calculated with the indicated classification of the series.

For other cases involving two or more series, working formulas may be derived for estimating the optimal theoretical size of the class interval.

HERBERT A. STURGES

Washburn College

COMMENT ON "A FORMULA FOR FREIGHT RATES"

The note on a formula for freight rates in this JOURNAL for September, page 416, is of interest in suggesting that a hyperbola fits existing freight schedules better than a parabola, but in some respects the note is misleading. A table of ton-mile costs for varying distances is quoted from the Nimmo Internal Commerce Report of 1876 as showing that "haulage costs do not increase at a fixed ratio, but the ratio of increase becomes smaller as mileage increases." The figures quoted show a constant ratio. They are derived from the assumption of a constant haulage factor of 9 mills per ton-mile combined with a fixed terminal cost of 31.62 cents per ton. Thus for 10 miles this gives a total rate of 40.62 cents, or 4.062 cents per ton-mile, as in the table quoted. For 100 miles, the total rate is $1.2162 and the rate per ton-mile is 1.216 cents, as in the table; and for 1,000 miles the total rate is $9.3162, and per ton-mile 0.93162 of a cent, rounded off at 0.932 in the table. In short, when we wish to construct rate scales on the basis of the cost of the service, we should use the formula for a straight line. There is no evidence that, after excluding from consideration the terminal costs and other factors wholly independent of distance, the road haul cost is any less for the fourth, fifth, or sixth hundred miles than it is for the first or second. It has been suggested that the shorter hauls are more expensive because the lightly loaded local trains are relatively more numerous for such distances. But this is not a satisfactory basis for saying that road haul costs decline with distance.

The true basis for a diminishing rate of progression for the road haul factor in the freight charge is in commercial considerations of what the traffic will bear. A lower profit per ton on the long haul widens the area of competition. The real difficulty in constructing rate scales is not in finding a curve that will best fit a given number of rates, but it is in determining what the key rates should be.

M. O. LORENZ

Washington, D. C.

PROGRESS OF WORK IN THE CENSUS BUREAU

A QUICK AND INEXPENSIVE CITY CENSUS

Under the supervision of Edward W. Koch of the Bureau of the Census, the city of Louisville, Kentucky, has recently taken a census through the agency of volunteer enumerators, working without compensation.

The preliminary work was begun December 1, 1925, with the aid of a very active local committee composed of the Mayor, and representatives of the Board of Trade, Bar Association, Real Estate Board, Publicity League, and various Luncheon Clubs. The first appeal for volunteer workers was made at

the luncheon of the Rotary Club on December 3. This was followed by appeals to the other civic bodies, and through the newspapers to the public in general. The response was very gratifying, and on December 9 the organization was substantially complete as follows:

4 Chief Inspectors, each in charge of 3 wards,
59 Inspectors, each in charge of from 10 to 15 enumerators,
700 Enumerators.

The schedule was simple, providing only for the address, name, relationship to head of the family, sex, and color.

The enumeration started promptly on Thursday, December 10, and by Saturday probably 90 per cent of the work was completed, but delays in scattered districts prevented the final completion until Sunday, December 20.

The returns were checked to the street portion of the city directory, counted, and tabulated by about 50 volunteer workers provided by banks, trust companies, and others, the greater part of this work being done at night.

The population, 305,935, was announced on December 24, just three weeks after the first appeal for workers was made.

This is probably the first time in the United States that a census by unpaid volunteers has been taken in a city of considerable size. The entire cost of the work was in the neighborhood of $1800, this sum including the expenses of the local committee which visited Washington to make arrangements. The only salaries paid by the city were those of two stenographers, the salary of Mr. Koch being paid by the Bureau of the Census.

CENSUS OF AGRICULTURE

The first state bulletin (New Hampshire) presenting the results of the 1925 census of agriculture has been published, and several others are in the hands of the printer. These bulletins, which are of octavo size, contain five general tables giving, by counties, the data on (1) farms and farm acreage; (2) farm values, mortgage debt, and miscellaneous items, including farm expenditures, farm facilities (tractors, radios, and silos), purchases and sales through coöperative organizations, and kind of road (concrete, macadam, gravel, etc.) on which farm is located; (3) livestock and livestock products; (4) value, acreage, and production of crops; and (5) farm population.

CENSUS OF INSTITUTIONAL CLASSES, 1923

The report on the Insane, under the title of "Patients in Hospitals for Mental Disease," is in galley proof, as are also the reports on Prisoners and on Children under Institutional Care. A short bulletin (12 pp.) presenting summary statistics for Prisoners was issued in December. Copy for the reports on the Feeble-minded and Epileptic was sent to the Government Printing Office in October.

MONOGRAPHS

Mr. Woodlief Thomas of the Federal Reserve Board is assisting Dr. Edmund Day in completing the monograph on Growth of Manufactures. The mono-

graphs on Immigrants and Their Children by Niles Carpenter, and on Farm Population by Leon E. Truesdell are in galley proof; and copy for the monograph on Wages in Manufacturing Industries by Paul F. Brissenden has gone forward to the Government Printing Office.

Arrangements have been made for an additional monograph based on the population census having as its subject the ratio between number of children and number of possible mothers. This monograph will be contributed by Dr. Warren S. Thompson, of the Scripps Foundation for Research in Population Problems.

FINANCIAL STATISTICS OF STATES AND CITIES

Preliminary printed bulletins covering the year 1924 have been published for each state and for each of the 248 cities that had over 30,000 population in 1920. The complete 1924 report on the states was sent to the Government Printing Office in November, 1925; and that on the cities in February, 1926. Meantime the canvass for 1925 has been begun and preliminary press summaries are being issued as fast as the reports for the individual states and cities are received and tabulated.

VITAL STATISTICS—EXTENSION OF THE REGISTRATION AREA

Encouraging progress is being made in the "drive," to bring every state into the registration area before 1930. Of the 48 states, 41 are now in the death registration area and 33 in the birth registration area, these two areas comprising respectively 91.6 per cent and 75.9 per cent of the population of the United States. West Virginia and Alabama were admitted to the registration area for 1925. This leaves 7 states to be added to the death registration area and 15 to the birth registration area within the next four years.

To promote this drive, Dr. Wilmer R. Batt, for twenty years State Registrar of Pennsylvania, has been appointed a special agent of the Bureau to pass on to the various states the results of his long experience. The Vital Statistics Section of the American Public Health Association, through a special committee of which Dr. Louis I. Dublin is chairman, has also taken up the fight and is cooperating in every possible way with the Bureau and with Dr. Batt in the efforts to reach the "vital statistics goal."

J. A. H.

REPORT OF THE PRESIDENT OF THE SOCIAL SCIENCE RESEARCH COUNCIL FOR THE YEAR 1925[1]

The Social Science Research Council was organized in 1923 by concurrent action of national associations interested in social research. This group at first included the American Economic Association, the American Sociological Society, the American Political Science Association, and the American Statistical Association. During the year 1925 the membership of the Council was increased by the

[1] This constitutes the report of the Statistical Delegation on the Committee of the Social Science Research Council, mentioned in this issue of the JOURNAL, on p. 92.

addition of representatives from the American Psychological Association, the American Anthropological Association, and the American Historical Association. The membership of the Council is at present as follows:

American Economic Association
Horace Secrist, Northwestern University
John R. Commons, University of Wisconsin
George E. Barnett, Johns Hopkins University

American Political Science Association
Charles E. Merriam, University of Chicago
Robert T. Crane, University of Michigan
A. B. Hall, University of Wisconsin

American Sociological Society
F. Stuart Chapin, University of Minnesota
William F. Ogburn, Columbia University
Shelby M. Harrison, Russell Sage Foundation

American Statistical Association
W. F. Willcox, Cornell University
Edmund E. Day, University of Michigan
H. L. Rietz, State University of Iowa

American Psychological Association
Robert S. Woodworth, Columbia University
Robert M. Yerkes, Yale University

American Anthropological Association
Clark Wissler, Yale University
Fay-Cooper Cole, University of Chicago
W. D. Wallis, University of Minnesota

American Historical Association
Guy S. Ford, University of Minnesota
William E. Dodd, University of Chicago
Arthur M. Schlesinger, Harvard University

These seven organizations are now brought together for the purpose of promoting the interest of scientific research in the field of social inquiry, particularly in cases where problems overlap the boundaries of one or more of the special fields concerned. It is believed that with the seven organizations now united it will be possible to advance the prospects of social science by the study of methods of social research, by consideration of special problems, and by coördination of scattered types of inquiry otherwise independent and isolated.

During the year 1925 the Council appointed a special Committee on Problems and Policy for the purpose of considering certain special questions already before the Council, as well as others, and of canvassing the general policy to be followed by the Council. The Committee on Problems and Policy held a ten day's session at Dartmouth during the summer and considered at length the work of the Council in general and a number of specific problems in particular. As a result of

this conference the Council decided to organize a standing committee known as the Problems and Policy Committee to consist of six members chosen by the Executive Committee for a term of three years. This Committee, under the general direction of the Council, will have power to devise and recommend research problems referred to it by the Council, and any other problems as the Committee may see fit to recommend. The Committee will ordinarily deal with each of the following aspects of the problems considered:

1. The practicability of the problem for scientific investigation.
2. Adequateness and appropriateness of the technical plans and budget involved.
3. The selection of the personnel for the supervision of the problem.

The Committee will have power to appoint special advisory committees, of ordinarily not more than five, to consider the formulation of a problem, to analyze the problem into parts susceptible of scientific treatment, to study the character and scope of the investigations which seem desirable, and to suggest agencies whose coöperation can profitably be enlisted in the work. This Committee now consists of the following members:

Professor A. B. Hall, University of Wisconsin (Chairman)
Professor Edwin F. Gay, Harvard University
Mr. Shelby M. Harrison, Russell Sage Foundation
Professor Clark Wissler, Yale University
Dr. H. G. Moulton, The Institute of Economics
Professor R. S. Woodworth, Columbia University

The Committee recommended and the Council approved the setting up of committees carrying on research in the field of alcoholism, in the Negro problem, in the study of crime, in the field of agricultural economics, and in certain significant phases of social and industrial relationships.

On the recommendation of the Committee, the Council at its last meeting also adopted the following general policies in respect to research:

A. Ordinarily it will be the policy of the Council not to undertake investigation directly other than preliminary studies.
B. Ordinarily the Council should deal only with such problems as involve two or more disciplines.
C. Generally it should be the policy of the Council to serve only as a clearing house in matters of research in the social science field.

Furthermore it was determined by the Council to undertake the gathering of pertinent information concerning research projects, personnel, funds and endowments available for research. It was understood that the Council would coöperate with any other agencies interested or engaged in similar enterprises in overlapping fields.

It is hoped that the administration of the Council's projects and problems will be covered by adequate financial arrangements for this purpose. During the year 1925 a grant was made to the Council for the expenses of general administration by the Russell Sage Foundation, but a new budget is being prepared covering the work of the Council and the Problems Committee which it is hoped may be favorably acted upon in the near future.

During the year 1925 funds were made available to the Council for the purpose of awarding Fellowships to advanced students desiring to carry on social research in the field of the social sciences, broadly construed. Broadly speaking, these Fellowships correspond to those awarded by the National Research Council. Evidence of exceptional ability in research must be presented by each applicant, together with a definite outline of a project giving promise of scientific accomplishments. The terms of the Fellowship may range from several months to as much as two years, depending upon the character and requirements of the problem. The work of the Fellows is subject to the supervision of the Council's Committee on Fellowships, of which Professor Wesley C. Mitchell is Chairman and Professor F. S. Chapin, of the University of Minnesota, Secretary. A substantial fund to cover these Fellowships for a period of five years has been set aside by the Laura Spelman Rockefeller Memorial. In the year 1925 the sum of $49,000 was available for this purpose. (The first awards of the Council are published in the "Announcement of Research Fellowships in the Social Sciences by the Social Science Research Council.")

During the last year the Committee on Human Migration, of which Dean Abbott is Chairman, continued the development of its projects. One unit of the plan was undertaken by the National Bureau of Economic Research under whose general direction Professor Jerome of the University of Wisconsin was engaged in the study of the relation of the mechanization of industry to migration. This project was continued during the year 1925–26 and will be completed by July 1, 1926.

The Committee also undertook a statistical study of the basic movements in migration in recent times under the direction of Professor Walter Willcox of Cornell University. In coöperation with the National Research Council's Committee on Human Migration (of which Professor Stratton is Chairman) a comprehensive plan is now being worked out and it is hoped that the plan may be completed within a short time and its execution vigorously pushed forward. The coöperation of the Committees from the two Councils offers an excellent example of the possibilities and also the difficulties of bringing about successful coöperation between those interested in the social implications of natural science and those interested in social science.

The Committee on International News and Communication, of which Mr. Walter S. Rogers is Chairman, continued the development of its program during the year 1925. An interesting offshoot of the work of this Committee is the establishment in 1925 of an Institute of Current World Events, a foundation which will make possible a detailed study of and reporting on current social events in a wide range of nations. This foundation, of which Mr. Rogers is Director, will undertake to develop personnel for the purpose of studying questions of news and public opinion in different parts of the world and of reporting their findings in the United States by means of articles, addresses and discussions. This project is now just beginning, but is already financed on a scale sufficiently broad and generous to make it possible to test out its possibilities. While this result was not anticipated when the Council created the Committee, it illustrates the possibilities of indirect development in collateral fields.

The Committee on Indexing and Digesting of the session laws of the various states, of which Professor Joseph P. Chamberlain of Columbia University is in charge, has continued its activities during the year 1925 and has made substantial progress. An appropriate bill has been carefully drawn and the whole question will come before the House Judiciary Committee during the coming winter. It is hoped that it will be possible to make progress with the financing of this very significant project. Through the efforts of the Committee the support of a large number of organizations has been secured and there is every reason to believe that the work of the Committee will be successful in the near future. This project, if carried through, would constitute an achievement of very great significance in the practical study of American legislation.

The Committee on Social Science Abstracts, of which Professor F. S. Chapin of the University of Minnesota is Chairman, is still engaged in the development and financing of its plan. The Committee's activities during the year 1925 include:

(1) The preparation of sample abstracts of social science articles drawn from the fields of Anthropology, Economics, Political Science and Sociology. This material will be published in the form of a dummy for distribution among members of the Social Science Societies in order to ascertain the interest in a possible Journal or Review of Social Science Abstracts and to determine what support may be obtained in the form of individual subscriptions for such a publication.

(2) Promising contacts have been established with several publishing houses regarding the publication of a Journal or Review of Social Science Abstracts as soon as a budget and editorial arrangements can be worked out. With assurances of some subscriptions and a moderate endowment the Committee believes that a publishing house will be found willing to undertake the publishing of this Journal.

(3) The Committee has also undertaken to obtain a subvention to establish a Journal or Review of Social Science Abstracts.

The Committee on the Survey of Social Science Agencies, of which Professor Horace Secrist is Chairman, has continued its consideration of the plan for a study of social research agencies, with special reference to the technical methods employed, and with the hope both of developing closer coördination of social research projects and of aiding in the evolution of more scientific approach to social problems. This Committee, one of the first organized by the Council, has been reconstructed this year and is prepared to pursue its objectives more effectively.

On the whole the Council has made substantial progress in 1925, both in the direction of more effective organization and in dealing with specific types of problems. It is the hope of the members of the Council that it may be increasingly useful to students of social science and that the various constituent organizations and their respective members may find it helpful in the organization and development of technical social research. The Council is in an experimental state, and suggestions for making undertakings and methods more valuable to the social sciences or to those interested in the social implications of natural science are welcomed.

CHARLES E. MERRIAM, *President*

MISCELLANEOUS NOTES

Chicago Statistical Dinner.—A dinner meeting of members of the American Statistical Association in and about Chicago was held at the City Club of Chicago, December 16, 1925. The general topic for discussion was Statistics in the Service of Industry. The leading papers were presented by J. M. Stahr, of the Western Electric Company, G. L. Seaton, of the Illinois Bell Telephone Company, and E. N. Wentworth, of Armour's Livestock Bureau.

Mr. Stahr, speaking on the subject of Statistical Methods of Standardization and Control, outlined some of the ways in which, at the Hawthorne Plant of the Western Electric Company, statistical methods have been applied to the administrative problems of an industrial corporation employing, at times, some 40,000 persons, and manufacturing about 110,000 separate kinds of parts. He sketched the procedures followed in the statistical control of production, cost, and personnel, dwelling particularly on the sampling problems involved in the inspection of the product, on the problem of adequate quantitative determinations of defect in terms of the resultant loss, and on devices for simplifying the task of valuing a stock of parts represented by entries on 70,000 stock cards.

Mr. Seaton, discussing Trends and Seasonal Factors in the Telephone Industry, analyzed some of the data gathered by the Commercial Engineering Department of the Illinois Bell Telephone Company, and explained the methods which have in practice been applied to them. In the study of trends, he said, simple methods of curve-fitting had been chosen, on the ground that more elaborate formulae seemed to yield no appreciable practical gain. Straight-line trends, sometimes on semi-logarithmic paper, had been preferred. It had proved difficult to determine any typical length of the cycle in the telephone business in Chicago. The seasonal fluctuation, on the other hand, was very marked and characteristic. Mr. Seaton exhibited a series of charts. Conspicuous among these was a chart of removal orders received, by months, in which the constant recurrence of an extreme peak in April and a secondary peak in September vividly reflected the moving-day habits of the people of Chicago.

Mr. Wentworth showed another series of excellent charts to illustrate his remarks on Statistical Studies of the Livestock Supply. He commented upon the manifest inverse relation between receipts and prices, and upon the time lag between price movements and the movement of receipts in the cases of cattle, sheep and hogs. On the whole Mr. Wentworth found little correspondence between the cycles of livestock supply and the general business cycle; though for each kind of animal characteristic cyclical movements were observable, in the interpretation of which it seemed easy to recognize the influence of the time required in each case to breed and bring to marketable maturity a sufficient supply.

The general discussion which followed the papers centered about the applicability of so-called general business indices to the problems of particular industries. Mr. Stahr alone among the speakers thought that the general-business curve was closely paralleled by the curve of production in his own industry. He had noticed a decided correspondence between the course of pig-iron production and the changes in volume of orders for telephone equipment. He suggested in explanation that his company supplied users in many localities and probably in consequence felt the ups and downs of business in a very generalized way. Mr. N. E. Crump, of London, who was a guest at the meeting, remarked that American experience apparently supported the English saying that pigs and pig-iron were good signs of the state of business—though, as was

brought out by other comments, the supply of hogs in England is not, as here, obviously correlated with the crop of corn or of any other staple.

United States Bureau of Labor Statistics.—Field work has been completed in the collection of data concerning wages and hours of labor in coal mining, foundries and machine shops, lumber, and pottery. Summary figures have been published in different numbers of the *Monthly Labor Review* and bulletins are now in the course of preparation. Field work in the collection of similar data for slaughtering and meat packing is nearly completed, and agents are now collecting such data in the automobile industry.

Other agents are in the field collecting information concerning industrial health and recreation activities, as found in industrial establishments in the United States. A handbook relating to trade unions is in course of preparation.

Regular current work is being done in wholesale prices; retail prices of food, coal, gas, and electricity; employment; labor laws and decisions; industrial accidents; strikes and lockouts; digesting state and foreign publications relative to labor conditions, etc.

A limited study concerning the employment of convicts on roads will soon be started.

On December 17 President Coolidge sent to the Senate the name of Ethelbert Stewart to be Commissioner of Labor Statistics, this being a re-appointment. The appointment was confirmed December 21, 1925.

A Proposed Social Index.—On Wednesday evening, December 30, 1925, there was a gathering at the Golden Pheasant Restaurant in New York City, of about thirty persons interested in the development of a social index. A number of the members of the Association's Committee on Institutional Statistics were present. After considerable discussion it was decided to request the Board of Directors of the Association to appoint a committee to investigate both the desirability and the feasibility of establishing a social index. This matter will come up at the next meeting of the Board of Directors.

Washington statisticians are organizing the Washington Statistical Society as a branch or chapter of the American Statistical Association. At a meeting held at the Hotel Raleigh on Tuesday evening, January 19, a Constitution was adopted and officers were elected as follows: President, Joseph A. Hill; Vice-President, E. Dana Durand; Secretary, Dorsey W. Hyde, Jr. Dr. Durand read a paper on "International Statistical Organization." *The Washington Statistician*, published for the advancement of statistical work in Washington, has appeared as the informal organ of the Washington society.

The Bureau of Savings Research has been organized by savings banking interests. Its purpose is to study the development of savings and the problems of savings banking. Dr. W. H. Steiner is Director of the Bureau, with offices at 11 East 36 Street, New York City. Associated with him are three bankers, O. C. Lester, J. E. Baker, and W. E. Hallett, dealing respectively with savings development, investments, and method and accounting. The Advisory Council of the Bureau is composed of the following economists: H. P. Willis, H. G. Moulton, W. F. Gephart, W. C. Mitchell, and G. E. Roberts; and the Advisory Board of the following bankers: H. P. Gifford, A. E. Adams, J. M. Willcox, R. R. Frazier, and J. H. Puelicher.

Beginning January 1, 1926, the registration area for vital statistics in Canada will embrace the entire Dominion. Under the scheme in force, uniform legislation has been passed by each of the nine provinces based on a Model Bill, whilst the forms for the registration of births, deaths and marriages are supplied free of charge by the Dominion Bureau of Statistics. The latter also compiles the results according to a concerted scheme of tabulation and issues annual and monthly statements for the Dominion as a whole and for each of the provinces and the principal cities. Until the past year the Province of Quebec had not passed the necessary legislation, but this has now been remedied and a plan of collection devised that bids fair to place that Province in the forefront. A feature of the scheme is the payment of fifteen cents a form to the Collector of Vital Statistics.

The Eugenics Record Office of the Carnegie Institution of Washington, located at Cold Spring Harbor, Long Island, has issued a schedule for recording the physical development of American males and another for females. Each comprises eight charts on which are drawn an average developmental curve and standard deviations. The curve is on plotting paper, permitting one to record alongside of the standard curve, the curve of growth of an individual child. Persons interested in tracing the development of their children will receive a copy of either schedule by applying to the Eugenics Record Office.

HORACE G. WADLIN

Horace G. Wadlin of Reading, Massachusetts, former Librarian of the American Statistical Association and of the Boston Public Library, died at the Winchester Hospital on November 5 at the age of seventy-four. He was connected with the Massachusetts State Bureau of Statistics of Labor for many years and became its Chief in 1883, serving until his appointment as Librarian. He also served four terms in the Legislature. He was the author of many statistical and historical works.

GEORGE CLIFTON HALL

Dr. George C. Hall died at his home in Richmond, Virginia, October 12, 1925, in his fifty-eighth year. After receiving his degree of M.D., in 1890, he studied for ten years in Vienna and the Rotunda Hospital, Dublin, Ireland, and elsewhere in European and American medical centers. From 1904 to 1912 he was connected with the New York Life Insurance Company and the North American Life Insurance Company. He became Medical Director of the Life Insurance Company of Virginia in 1912, which office he held to the date of his death. He was a member of the American Medical Association, the Medical Directors Association of America, the American Statistical Association, and of the Virginia State, Tri-State and Southern Medical Societies.

Joseph V. De Porte, Ph.D., Director of the Division of Vital Statistics of the New York State Department of Health, has recently been elected to Fellowship in the Royal Statistical Society of England.

Dr. G. E. Harmon of the Department of Hygiene and Bacteriology of the School of Medicine of Western Reserve University is spending his sabbatical year (1925–1926) in study with Professor Karl Pearson at the Biometric Laboratory, University College, London.

Dr. John Rice Miner has been appointed Biometrician, with the rank of Associate Professor, in the Institute for Biological Research, Johns Hopkins University.

Mr. J. H. Riddle has left the Treasury Department of the United States to join the economic staff of the Agent General for Reparation Payments.

MEMBERS ELECTED SINCE DECEMBER, 1925

Adams, Harold E., Service Club (International Organization), Chicago, Ill.
Anderson, Montgomery D., 6261 Clemens Avenue, St. Louis, Mo.
Batchelor, James Harold, Arkansas Lumber Company, Guthrie, Okla.
Blackburn, Raymond F., University of Pittsburgh, Pittsburgh, Pa.
Brezee, Elizabeth L., Child Health Demonstration, Fourth Avenue and 10 Street South, Fargo, N. D.
Carroll, Professor Jean F., College of Commerce, Drake University, Des Moines, Iowa.
Collver, Clinton, Lyman D. Smith and Company, 44 Pine Street, New York, N. Y.
Conner, Paul, 754 Orange Street, New Haven, Conn.
Copeland, Professor Morris A., Cornell University, Ithaca, N. Y.
Cox, Professor Alonzo B., University of Texas, Austin, Texas.
Dameron, Wilbur Robbins, Farmers Loan and Trust Company, New York, N. Y.
Day, Oscar A., Department of Statistics, Agricultural Experiment Station, West Lafayette, Ind.
Dunlap, Walter H., Academy of Practical Economics, Washington, D. C.
Eberhard, George H., 360 Tremont Street, San Francisco, Calif.
Eckler, A. Ross, 27 Forest Street, Cambridge, Mass.
Evans, David, Jr., Rutter and Company, 14 Wall Street, New York, N. Y.
Fismer, Ester I., 1430 Massachusetts Avenue, Room 416, Harvard Economic Service, Cambridge, Mass.
Foss, Feodore F., Wheeling Steel Corporation, Wheeling, W. Va.
Frisbee, Ira N., 1185 North Alexandria Avenue, Los Angeles, Calif.
Gardner, Emmet G., Illinois Bell Telephone Company, Chicago, Ill.
Gauger, Caroline, Columbia University, New York, N. Y.
Graff, John H., Research Department, Brown Company, Berlin, N. H.
Grant, Florence A., 701 Washington Street, New York, N. Y.
Hall, Forest, School of Commerce, Denver, Colo.
Hansen, Wilfred I., The Utensils Company, Fort Wayne, Ind.
Higdon, John C., Business Men's Assurance Company, Kansas City, Mo.
Kreevoy, Edward P., Page Cabinet Company, Cambridge, Mass.
Latimer, Murray W., Bureau of Business Research, Harvard University, Cambridge, Mass.
Levin, Benjamin S., 2104 Chelsea Terrace, Baltimore, Md.
Lewis, Edward E., 67 Merrick Road, Baldwin, Long Island, N. Y.
Lingg, Claire, New York Heart Association, New York, N. Y.
Lowden, Honorable Frank O., Oregon, Ill.
Lundy, Elizabeth D., Barnard College, New York, N. Y.
McGoun, Professor A. F., 37 Upper Bellevue Avenue, Westmount, Quebec, Canada.
McIntosh, Albert J., Standard Oil Company of New York, New York, N. Y.
Mapes, Clarel B., Carr and McFadden, West Palm Beach, Fla.
Mattocks, Raymond L., 522 Fifth Avenue, New York, N. Y.
Mearns, Louise A., School of Business, Columbia University, New York, N. Y.
Mehl, Paul, United States Grain Futures Administration, Chicago, Ill.

Merrell, Margaret, School of Hygiene, Johns Hopkins University, Baltimore, Md.
Metzger, Dr. Hutzel, Division of Agricultural Economics, University Farm, St. Paul, Minn.
Millard, J. W., United States Department of Commerce, Washington, D. C.
Mittelman, Professor E. B., Oregon Agricultural College, Corvallis, Ore.
Newcomer, Professor Mabel, Vassar College, Poughkeepsie, N. Y.
Oberoutcheft, C. M., Cities Census Committee, 200 Fifth Avenue, New York, N. Y.
Ohse, Amelia E., 5 Pearson Avenue, Somerville, Mass.
Pell, John, 16 East 43 Street, New York, N. Y.
Pendergast, Roland B., 7 Craigie Circle, Cambridge, Mass.
Putnam, Persis, Department of Biometry and Vital Statistics, Johns Hopkins University, Baltimore, Md.
Reddy, Dorothy G., Columbia University, New York, N. Y.
Reeves, Francis B., Room 215, 50 Oliver Street, Boston, Mass.
Riedel, C. O., Brookmire Economic Service, Inc., Los Angeles, Calif.
Roman, Professor Frederick W., The Stable Money Association, 104 Fifth Avenue, New York, N. Y.
Ruppert, Frank D., Case, Pomeroy and Company, 60 Beaver Street, New York, N. Y.
Saffian, Sadie, Jewish Welfare Society of Philadelphia, Philadelphia, Pa.
Scadding, Simcoe C., The Bell Telephone Company of Canada, Montreal, Canada.
Schnitman, L. Seth, Survey of Current Business, United States Department of Commerce, Washington, D. C.
Sharbrough, Ralph B., The Great Atlantic and Pacific Tea Company, New York, N. Y.
Sheldon, Royal D., Southern California Telephone Company, Los Angeles, Calif.
Shields, D. Murray, Economic Research Department, Procter and Gamble Company, Cincinnati, O.
Smith, Hartley Le Huray, New England Power Company, Worcester, Mass.
Smithton, H. L., Statistical Bureau, Scripps-Howard Newspapers, Cincinnati, O.
Spafford, Ruth J., Columbia University, New York, N. Y.
Stahl, Archie E., 4406 Greenwood Avenue, Chicago, Ill.
Stern, Boris, Department of Commerce, Research Division, Washington, D. C.
Strudell, Fred D., 722 Chestnut Street, St. Louis, Mo.
Sweeting, Professor Charles L., Rhode Island State College, Kingston, R. I.
Szold, H. J., Lehman Brothers, 16 William Street, New York, N. Y.
Thomas, Brown A., Department of Research and Service, Security Trust and Savings Bank, Los Angeles, Calif.
Thompson, Donald S., Division of Analysis and Research, Federal Reserve Bank, San Francisco, Calif.
Tilton, J. W., Teachers College, Columbia University, New York, N. Y.
Upshall, Charles C., Columbia University, New York, N. Y.
Vaughan, Dr. Roger T., Cook County Hospital, Chicago, Ill.
Vinci, Professor Felice, Instituto Superiore di Scienze, Venice, Italy.
Watkins, Leonard L., 169 Kinsey Avenue, Cincinnati, O.
Watkins, Ralph J., Bureau of Business Research, Ohio State University, Columbus, O.
Wilson, Edward F., The J. G. White Management Corporation, 33 Liberty Street, New York, N. Y.
White, Percival, 175 Fifth Avenue, New York, N. Y.

PROCEEDINGS OF THE EIGHTY-SEVENTH ANNUAL MEETING OF THE AMERICAN STATISTICAL ASSOCIATION

NEW YORK, N. Y., DECEMBER 28–30, 1925

PROGRAM

MONDAY, DECEMBER 28

2.30 P. M. Hotel Pennsylvania, Grand Ball Room. Joint Session with American Economic Association and with American Association for Labor Legislation

Presiding Officer—Sam A. Lewisohn, New York City

The Movement of Real Wages and Its Significance

Papers:

1. "The Best Measure of Real Wages"
   Alvin H. Hansen, University of Minnesota
2. "The Economic Significance of Changes in Real Wages"
   Paul H. Douglas, University of Chicago
3. "Real Wages and the Control of Industry"
   Alvin S. Johnson, The New Republic, New York City

Discussion:

1. Paul F. Brissenden, Columbia University
2. D. A. McCabe, Princeton University
3. George Soule, Labor Bureau Inc., New York City
4. Magnus W. Alexander, National Industrial Conference Board, New York City

8.00 P. M. Columbia University, McMillan Auditorium. Joint Session with American Sociological Society

Presiding Officer—Franklin H. Giddings, Columbia University

Presidential Addresses

1. "Position as a Sociological Concept"
   Robert E. Park, University of Chicago, President American Sociological Society
2. "The Place of Statistics in Undergraduate Training"
   Robert E. Chaddock, Columbia University, President American Statistical Association

9.30 P. M. Columbia University, Faculty House, 400 West 117th Street. Reception by President Nicholas Murray Butler of Columbia University and Mrs. Butler

TUESDAY, DECEMBER 29

9.00 A. M. Hotel Pennsylvania, Southeast Ball Room

Business Meeting—Election of Officers, Amendments to Constitution, and Official Reports

10.00 A. M. Hotel Pennsylvania, Southeast Ball Room. Program under the auspices of the Association's Committee on Governmental Labor Statistics

Presiding Officer—Mary Van Kleeck, Russell Sage Foundation

Should Governmental Bureaus of Labor Statistics Collect Current Statistics of Labor Turnover and in What Form Should the Data be Obtained?

1. "Significance of Statistics of Labor Turnover"
   Anna Bezanson, University of Pennsylvania
2. "The Best Measure of Labor Turnover"
   Paul F. Brissenden, Columbia University

3. "Measurement of Labor Turnover as a Problem of Management"
   William A. Berridge, Metropolitan Life Insurance Co.
4. "The Place of Statistics of Labor Turnover in a Program of Governmental Labor Statistics"
   Ethelbert Stewart, U. S. Bureau of Labor Statistics

Discussion: Opened by Leonard W. Hatch, New York State Department of Labor

2.30 P. M. Hotel Pennsylvania, Southeast Ball Room

Presiding Officer—Louis I. Dublin, Metropolitan Life Insurance Co.

Standards and Requirements in Statistics?
What is Expected of Statisticians by Employers?
How Shall Statisticians be Trained?

1. "The Field of Business and Economic Research"
   Donald R. Belcher, American Telephone & Telegraph Co.
   Carl Snyder, Federal Reserve Bank of New York
2. "Public Health, Medicine, Biology, and Insurance"
   Edgar Sydenstricker, U. S. Public Health Service
   Edwin W. Kopf, Metropolitan Life Insurance Co.

6.30 P. M. Aldine Club, 200 Fifth Avenue. Dinner Meeting

Presiding Officer—James W. Glover, University of Michigan

Training in Statistical Methods in Colleges and Universities (Viewpoint of Teachers of Statistics)

1. "Content and Purpose of Training in Elementary and in Advanced Statistics"
   H. C. Carver, University of Michigan
   Willford I. King, National Bureau of Economic Research
2. "Mathematical Background for Statistical Training in—
   (a) "Elementary Methods
   (b) "Advanced Methods and Professional Research"
   H. L. Rietz, Iowa State University
   A. R. Crathorne, University of Illinois
3. "What is Being Done in Colleges and Universities"
   James W. Glover, University of Michigan
   (A questionnaire has been sent out and a digest of the replies made by the Committee on Education and Professional Standards of the Association, of which Professor Glover is Chairman)

WEDNESDAY, DECEMBER 30

9.00 A. M. Hotel Pennsylvania, Southeast Ball Room

Business Meeting—Reports of Committees and Other Business

10.00 A. M. Hotel Pennsylvania, Small Ball Room. Joint Session with the American Association for Labor Legislation

Presiding Officer—Mary Anderson, Women's Bureau, Washington, D. C.

Are Accidents Increasing? How Can They be Prevented?

Papers:

1. "The Accident Rate"
   Leonard W. Hatch, New York State Department of Labor
2. "Progress in Accident Prevention"
   Lewis A. De Blois, National Safety Council, Manager, Safety Division, E. I. DuPont De Nemours Co.

3. "Organization for Prevention of Accidents"
R. H. Lansburgh, Secretary of Labor and Industry, Pennsylvania

Discussion:

1. Louis I. Dublin, Metropolitan Life Insurance Co.
2. Julius H. Parmelee, Bureau of Railway Economics, Washington, D. C.
3. Ethelbert Stewart, U. S. Bureau of Labor Statistics

2.30 P. M. Hotel Pennsylvania, Southeast Ball Room

Presiding Officer—Leonard P. Ayres, Cleveland Trust Co.

Economic and Business Research

Papers:

1. "The Statistical Allocation of Joint Costs"
W. L. Crum, Committee on Economic Research, Harvard University
2. "The Construction of an Index Number of Bond Yields"
Frederick R. Macaulay, National Bureau of Economic Research
3. "Price Displacement and Its Measurement"
Frederick C. Mills, National Bureau of Economic Research

Discussion:

1. Norman Crump, Compiler of the *Financial Times' Index* Number of Wholesale Prices, London, England
2. Max O. Lorenz, Interstate Commerce Commission, Washington, D. C.
3. Henry Schultz, U. S. Children's Bureau, Washington, D. C.

MINUTES OF THE ANNUAL BUSINESS MEETING

The first session of the annual business meeting of the American Statistical Association was held in the Hotel Pennsylvania, New York City, on Tuesday, December 29 at 9 A.M. Both the report of the Secretary and the report of the Treasurer were presented by the Secretary-Treasurer. Each report was accepted and ordered published.

The amendment to the last sentence of Article VIII of the Constitution of the Association, which was passed at the annual meeting in 1924, was finally adopted. The last sentence of the Article as amended reads:

> The President, Vice-Presidents, Secretary, Treasurer, Counsellors and the person or persons holding the presidency of the Association during the two preceding years shall form a Board of Directors for the government of the Association, three of whom shall constitute a quorum at any meeting regularly convened.

The motion was carried to amend Section 5 of the By-Laws to read:

> The Board of Directors shall, as soon as practicable after election, appoint such District Secretaries as is deemed necessary. Each District Secretary shall serve for one year or until his successor is appointed.

The Secretary, in compliance with the unanimous vote of those present, cast a ballot of the Association for the following officers placed in nomination by the Nominating Committee: President, Leonard P. Ayres; Secretary-Treasurer, Willford I. King; Editor of the JOURNAL, William F. Ogburn; Vice-Presidents, Truman L. Kelley, Frederick C. Mills, Raymond Pearl; Counsellors, Allyn A. Young, Robert E. Chaddock, N. I. Stone.

In view of the fact that the Editor had signified in a letter to the President that he did not desire re-election, a resolution was passed authorizing the Board of

Directors to fill the vacancy in case that the Editor, William F. Ogburn, should persist in declining the election.

The meeting adjourned.

The second session of the business meeting of the Association was held at the Hotel Pennsylvania, New York City, on Wednesday, December 30, at 9 A.M.

The report of Dr. Frank A. Ross, Acting Editor of the JOURNAL was read and accepted.

The report of the Committee on Governmental Labor Statistics was made by the Chairman, Miss Mary Van Kleeck. The motion was made and carried to accept and file the report.

The report of the Finance Committee was made by the Chairman, Dr. Edmund E. Lincoln. This report was also accepted and ordered filed.

The report of the Membership Committee was made by the Chairman, Mr. Edwin W. Kopf. It was accepted and ordered to be filed.

The report of the Committee on Institutional Statistics was made by the Chairman, Dr. Horatio M. Pollock. This report was accepted and ordered filed.

The report of the Committee on the Handbook of Social Sciences was made by Miss Mary Van Kleeck, the Chairman. The Committee requested an appropriation of the sum of $250.00 for current expenses. The motion was made and carried to accept the report but to refer the request for an appropriation to the Board of Directors for its consideration.

The report of the Statistical Delegation on the Committee of the Social Science Research Council was presented by Dr. Horace Secrist. The motion was made to adopt the report and to instruct the editor to print all the essential parts of the same. The motion was carried.

The motion was carried to amend the By-Laws by inserting a new Section 7 and making Section 7, Section 8. The new Section 7 is as follows:

> The Board of Directors shall, after consultation with the Director of the Census, fill all vacancies in the American Statistical Association delegation to the Joint Advisory Committee on the Census.

The following resolutions were adopted:

1. The Secretary of the Association is instructed to extend to the President, Robert E. Chaddock, the thanks of the members for his efficient service during the year and especially for his part in preparing the interesting and well balanced program presented at the Annual Meeting.
2. The Secretary is instructed to express on behalf of the Association, appreciation of the courtesy of President Nicholas Murray Butler and Mrs. Butler, of Columbia University, in entertaining the members of the Association.
3. The Secretary is instructed to extend the thanks of the Association to the Russell Sage Foundation for the marked assistance given by that Foundation to the work of the Association during the year, especially in connection with the preparation of the report of the Committee on Governmental Labor Statistics.

The meeting adjourned.

WILLFORD I. KING, *Secretary*

Report of the Secretary

Attention should be called to the fact that the membership record for December 1, 1925 is on a somewhat different basis from that of preceding years, only those being included in the list who were paid up at that time. Under these circumstances the gain of 208 members must be considered a very satisfactory one.

Membership, December, 1924			1,135
Additions during 1925		337	
Deductions during 1925			
Deaths reported during 1925	9		
Resignations during 1925	46		
Dropped because unable to locate	8		
Dropped for non-payment of dues	66	129	
Net gain in membership			208
Membership December 1, 1925			1,343
Besides members, we have:			
Subscribers, mostly libraries		347	
Domestic exchanges		28	
Foreign exchanges		76	451
Total members, subscribers, and exchanges			1,794

The following deaths have been reported during the year: Dr. Otto Eichel, David P. Fackler, Charles S. Fairchild, Dr. Louis Guillaume, Professor Joseph French Johnson, Logan G. McPherson, H. W. Shute, Horace G. Wadlin, and Honorable George Peabody Wetmore.

Four numbers of the JOURNAL, containing 604 pages in all, have been issued during 1925. They compose Volume XX. A total of 24 pages of paid advertising appeared in the four issues.

Five special meetings and dinners were held in New York City during the year. The attendance was respectively 99, 412, 103, 145, and 147. Two such meetings were held in Philadelphia, one formal and a dozen or more informal meetings in Detroit, six meetings in Los Angeles, and one meeting each in Washington, Chicago, and Cleveland.

The employment clearing house has been reasonably successful in putting members in touch with satisfactory positions. It is at the service of all members who desire to make use of it.

Miss Rennette Seltzer resigned on October 7, 1925 after having given a year of admirable service as assistant to the Secretary. Her place is now filled by Miss Abby Furbeck.

WILLFORD I. KING, *Secretary*

Report of the Treasurer Covering the Period December 15, 1924 to December 1, 1925

Receipts

Cash in checking account December 15, 1924	$292.07		
Bowery Savings Bank balance, December 15, 1924	3,000.00		
Total		$3,292.07	
Miscellaneous Cash received December 15 to December 31, 1924		61.08	
Interest on savings account, last half of 1924		60.00	
Cash Balance December 31, 1924			$3,413.15
Membership dues		6,893.80	
Sales, single copies and sets of JOURNAL		791.95	
Sales, Memorial Volume:			
Sales direct to members	105.60		
Royalties from Macmillan Company	113.47		
Total		219.07	
Advertising in JOURNAL		491.00	
Subscriptions to the JOURNAL		1,633.20	
Dividends on B. & A. R. R. stock		148.75	
Corporation Membership dues		500.00	
Gain from New York meetings		514.51	
Interest		104.74	
Life Membership dues		200.00	
			11,497.02
Total Receipts			$14,910.17

Expenditures

Net Cost of JOURNAL:			
Handbook and Index December 1924	$358.99		
December 1924 JOURNAL	1,032.33		
March 1925 JOURNAL	1,168.90		
June 1925 JOURNAL	1,117.76		
September 1925 JOURNAL	1,438.64		
Total		$5,116.62	
Postage, stationery and miscellaneous printing		1,353.50	
Salaries		3,112.06	
Rent		312.00	
Telephone and telegraph charges		61.96	
Furniture		111.04	

Annual and District Meetings, traveling expenses	$221.44	
Miscellaneous expenses	78.03	
Purchases of JOURNALS from members	173.65	
Total Expenditures		$10,540.30
Balance in Bowery Savings Bank	3,356.03	
Cash on hand, November 30, 1925	1,013.84	
		4,369.87
Total		$14,910.17
Total Cash Expenditures Outside of New York Meetings		$10,540.30
Expenses of New York Meetings	$2,146.88	
Miscellaneous adjustments	65.50	
		2,212.38
Total Expenditures		$12,752.68

Functional Apportionment of Expenditures

New York Programs, Annual and District Meetings	$3,215.28	
JOURNAL	7,073.68	
Employment Clearing House	545.06	
New Members	1,067.17	
Miscellaneous	851.49	
		$12,752.68
Total Expenditures		$12,752.68

On December 1, 1925 the Association possessed in addition to stationery and miscellaneous supplies the following assets.

17 Shares of Boston and Albany stock, market value December 1, 1925	$2,703.00
Furniture—carried at	299.08
Cash	4,369.87
Bills Receivable	183.00

7,278 JOURNALS of various issues.

187 Memorial Volumes.

There were no bills outstanding.

Up to date returns from the Memorial Volume have failed to cover expenses by $57.38.

WILLFORD I. KING, *Treasurer*

Report of the Acting Editor

In the absence of the Editor-in-Chief, Professor Ogburn, the task of reporting on the JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION has devolved upon the Acting Editor.

A decade ago the average number of yearly pages was approximately 350. Today it is approximately 600. Thus in 1916 the number was 470, in 1917, 402 and currently 308, 331, and under Professor Ogburn's administration 531, 518, 560, 520, 562 and 598. In 1925 the average size of issue was 150 pages. This seems as large as it is desirable that an issue should be. Indeed, it may have run somewhat beyond the optimum limit.

During this time the character as well as the bulk of the JOURNAL has been modified. Due to what has seemed a real demand, the section on Notes has been materially expanded. With the increasing publication of volumes on statistical method and those concerned with its application, the section on Reviews and on Current Literature under the direction of Dr. Leo Wolman has become an important and highly useful part of each issue. It has become a fixed policy to list by name the current additions to our membership list.

The local meetings of the Statistical Association groups in the larger cities have brought out papers of considerable worth, both in the development of technique and in its application. These have been broadcast through the JOURNAL either as complete papers or as condensed descriptions of entire meetings. Effort has also been made to keep our membership informed as to the developments in the more important data-gathering bureaus of the Federal Government.

The JOURNAL has become the forum not only of those interested in economic and social problems, but of those who are creating new methodology in the general field of statistics, particularly mathematical methodology.

Over the past decade the increasing importance of statistics in specialized fields in the social sciences has tended to force upon the older and newer journals in these specialties more and more quantitative treatment. Today writers who have completed statistical studies in the fields of public health, of psychology, or the like, submit their manuscripts to journals in these fields. There is a growing realization among authors that these publications will reach the specialized audiences more fully than will the JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION. In consequence the proportion of problem-type articles received by this office is constantly diminishing.

It seems improbable that the bulk of the average issue of the JOURNAL should be increased. On the other hand, the supply of material for publication has expanded more rapidly than the possibilities of publication. This is particularly true of two types—those involving large amounts of mathematical formulation, and those containing extensive tabular material. These latter constitute by far the most expensive element in printing costs, and, it is believed by some, are of less interest to the general run of our readers. It has become a real task to select wisely from among this large mass of meritorious manuscript and to preserve proportions consistent with the interests of our readers and with the dignity essential to the organ of a learned society. Suggestion has been made that while

the number of pages per issue might be kept as at present or even be somewhat reduced, the number of issues per year could be increased to six or even more. During the coming year the Editor's office will have to determine upon a policy which, based upon these and many other considerations of importance, should render the JOURNAL even more useful to the readers than it is at present.

The Editor of the JOURNAL has two pleas to make: First, that the already large number of *manuscripts* submitted *be increased*, particularly those of the non-mathematical type and those with small amounts of tabular material, in order that the range of selection may be greater than at present; second, that the *funds available* for publication be made *larger* so that more papers of the type which should give the JOURNAL high rank among scientific periodicals may be included, and that other highly desirable developments may be realized.

FRANK A. ROSS, *Acting Editor*

Report of Committee on Governmental Labor Statistics

The primary interest of the Committee since its appointment in 1922 has been the improvement and extension of statistics of employment. These statistics consist of reports made monthly by employers to state and federal bureaus of labor statistics, showing the total number on a given payroll and the total wages paid. Six states, including New York, Wisconsin, Illinois, Massachusetts, Maryland, and California, now collect the data within their own borders and send reports to the Federal Bureau of Labor Statistics for inclusion in the national figures. Several other states, including Iowa, Pennsylvania, and Oklahoma, collect data on employment, but in these states coöperation with the Federal Bureau has not yet been arranged.

The Federal Bureau now covers in 1925 as many as 9,200 establishments in 52 manufacturing industries, employing nearly three million wage earners whose total earnings during one recent week were over seventy-five million dollars. The great extension which these figures represent is shown by comparison with the scope of employment statistics collected by the Bureau in 1921, at the time of the President's Conference on Unemployment, when about 700 establishments in 13 manufacturing industries, employing 500,000 wage earners, were covered. In addition to these figures on manufacturing, the Interstate Commerce Commission collects data on employment on railroads, and the Department of Agriculture on farms. Several of the states include branches of employment other than manufacturing. Despite this encouraging progress, however, the great majority of states have no information regarding employment within their own borders, although manufacturing plants in all states are contributing information to the Federal Bureau of Labor Statistics. The extension of collection of local data by the states, and the inclusion of industries not now covered, notably the building trades, are at present the two major needs in the effort to secure adequate statistics of employment in the United States.

The results of the work of the Committee have been embodied in a book now in press, which is to be published by the Russell Sage Foundation soon after the first of the year, with the title, *Employment Statistics for the United States, a Plan*

for National Collection and a Handbook of Methods Recommended by the Committee on Governmental Labor Statistics of the American Statistical Association. It is hoped that carefully planned publicity in trade papers and elsewhere may strengthen the work of existing bureaus along these lines, encourage appropriations by state legislatures, and stimulate other states to join in the plan of collection. The book is designed to be useful to governmental statisticians concerned with labor statistics, to all firms which make reports to governmental bureaus, and to students and teachers of statistics in colleges and universities, since it deals with the whole problem of collection of statistics on which little practical material is available for teaching purposes.

While the improvement and extension of statistics of employment will continue to be its primary interest, the Committee has given its attention during the year to another set of facts closely related to stabilization of employment, namely labor turnover. At the 1925 meetings of the Association a session was given on December 29 to this question:—Should governmental bureaus of labor statistics collect current statistics of labor turnover, and in what form should the data be obtained? The suggestion put before the meeting was that in a small number of industries an experiment be made by one or more governmental bureaus in the collection of the following information, each item to be secured for the payroll period nearest the 15th of each month, the same period for which employment statistics are now reported:—(1) Number on the payroll; (2) number hired during the payroll period; (3) number of separations during the same period, subdivided into lay-offs, discharges and resignations.

This proposal was fully discussed at a meeting of the Committee. It was agreed that the extension of employment statistics in other states and in a larger number of industries was more important than the collection of data on labor turnover, but that experiments in the collection of this information would throw light upon stability of employment and be a valuable supplement to employment statistics. The Committee therefore voted to recommend that a governmental labor bureau which might undertake such an experiment should collect monthly data on labor turnover, including at least the number of separations during the payroll period, and preferably also the number of accessions. It will be recognized that this modifies the proposal made at the session of the Statistical Association meetings in that it does not ask for the classification of causes of separation, since the Committee felt that only in the better organized plants was this information forthcoming and that an experimental collection would better be in simpler form.

Other topics to which the Committee will give attention during the coming year include the possibility of agreement upon a common base period for the several current indices of employment and production; the use of the employment statistics now being collected by governmental bureaus as a basis for dependable industrial accident rates; the possibility of the more adequate collection of wage statistics; and the best measure of employment in the building trades.

The membership of the Committee during the year was as follows: A. J. Altmeyer of the Wisconsin Industrial Commission, Charles E. Baldwin of the United States Bureau of Labor Statistics, Joseph A. Becker of the United States

Department of Agriculture, W. A. Berridge of Brown University, W. R. Burgess of the New York Federal Reserve Bank, R. D. Cahn of the Illinois State Department of Labor, R. H. Coats, Dominion Statistician of Canada, Frederick E. Croxton of Columbia University, J. F. Dewhurst of the Philadelphia Federal Reserve Bank, Ralph G. Hurlin of the Russell Sage Foundation, Don. D. Lescohier of the University of Wisconsin, M. O. Lorenz of the Interstate Commerce Commission, Eugene B. Patton of the New York State Department of Labor, Roswell F. Phelps of the Massachusetts Department of Labor and Industries, Walter W. Stewart of the Federal Reserve Board, Woodlief Thomas of the Federal Reserve Board, F. G. Tryon of the United States Geological Survey, and Leo Wolman of the New School for Social Research.

Respectfully submitted,

MARY VAN KLEECK, *Chairman*

Report of Finance Committee

The Finance Committee has made a campaign during the past year to increase the number of corporate memberships in the Association. While indirectly this effort may have served as "good will" advertising, no increase in membership has yet been secured, although several possible prospects are now in sight. It is recommended that each member of the Association who is engaged in industrial work make a definite personal attempt to secure his company as a member. This seems to be the only practicable method of extending the corporate membership, as the appeal to the average company must still be made on professional and educational grounds, rather than on the basis of actual services immediately rendered.

It is also believed that the income of the Association will be substantially augmented through the increase in the number of members during the coming year. The development of the Employment Clearing House, together with the active work of the Committee on Educational and Professional Standards, should make a more definite appeal to young and promising statisticians who have not in the past been active in the work of the Association. The increase in the circulation of the JOURNAL resulting from a larger membership should also lead to a better income from advertising.

While the finances of the Association are now in excellent shape, there is not yet a sufficient surplus to justify the payment of an honorarium to the Editor of the JOURNAL. However, it is hoped that the income will increase sufficiently to make it possible to take action on this matter before the end of the year.

It is also recommended that the desirability of a small permanent endowment be seriously considered by the membership of the Association. Such a fund would greatly add to the dignity of the organization, and would make it possible to encourage research along fruitful lines, and perhaps to secure for publication desirable articles which we are not now in a position to pay for.

Finally, the Finance Committee wishes to express its appreciation of the excellent work which has been done by Dr. King as Secretary-Treasurer of the Association during the past year, and by Dr. Kopf as Chairman of the Membership

Committee, both of whose efforts have helped to put the work of the Association on a more satisfactory financial basis, so that rapid progress should be made in the future.

EDMOND E. LINCOLN, *Chairman*

Committee on Membership

During 1925, the Committee continued its work of coöperating with the Secretary in securing desirable memberships. The organization of a Section of the Association in Pittsburgh was encouraged, and negotiations were continued for the incorporation in the membership of this Association of the Business Research Association of San Francisco, California. It is hoped that the membership of this latter Association would be willing to join the American Statistical Association as a District Section on the Pacific Coast. A joint meeting of the Business Research Association and of the American Statistical Association members on the Pacific Coast will be held shortly and it is hoped that at that time arrangements can be made for the merging of the two groups.

During the coming year the Committee hopes to invite to membership in the Association the teachers of statistics who replied to the questionnaires of the Association's Committee on Educational and Professional Standards. The Committee wishes to suggest to all members that they canvass their junior associates and send in applications for membership. The object of the Association is to include as many as possible of the persons in the United States and Canada who are interested in statistics.

E. W. KOPF, *Chairman*

Report of Committee on Institutional Statistics

Your Committee on Institutional Statistics has followed up the work begun in previous years by endeavoring to promote uniform systems of recording and reporting data in the several classes of charitable and penal institutions.

The Federal Census Bureau has rendered a great aid to this movement by adopting for use in its census of institutions for mental disease and the feebleminded the statistical systems promulgated by the National Committee for Mental Hygiene in coöperation, respectively, with the American Psychiatric Association, and the American Association for the Study of the Feebleminded.

Progress has been made in several fields during the past year.

The National Association for the Study of Epilepsy at its last annual meeting appointed a committee to plan a system of uniform records and statistics in institutions for epileptics. That committee has requested the aid of members of your Committee in the formulation of the proposed system. It is expected that the matter will be taken up in the near future.

The manual prepared by your Committee for use in recording and reporting statistical data in institutions for children has not yet been published. It was hoped that the Federal Children's Bureau would coöperate in the matter, but after long consideration, the Bureau advised your Committee that its printing

appropriation was insufficient for the publication of the manual. The Committee thereupon submitted the matter to the Child Welfare League of America. A favorable report relative to the manual was recently received from the executive director of the League and it is expected that the system, with some minor modifications that the Committee will make, will be published and introduced by the League.

Gains are slowly being made in the improvement of records in institutions for delinquents. Committees of the American Institute of Criminal Law and Criminology and the American Prison Association are working on the problem and will probably develop a practical scheme.

The Committee on Statistics of the National Probation Association has worked out an excellent scheme for uniform records and reports for the use of juvenile courts. The Committee is now extending its work to cover adult probation.

Your Committee through President Chaddock and in coöperation with committees of the American Psychiatric Association and the National Committee for Mental Hygiene, has made an appeal to Secretary Hoover of the Department of Commerce to call a conference of representatives of national organizations and State departments with representatives of the Census Bureau to consider the establishment of a division of institutional statistics in the Federal Census Bureau. This matter was referred to in the report of your Committee last year and this Association went on record as favoring the project. The aim is to secure a comprehensive annual statistical review of the population of institutions throughout the country, and to secure the publication of such a review within a year of the enumeration date. This matter of far-reaching importance should enlist the hearty support of the members of this Association.

It is a matter of regret that most of the reports of the Federal census of institutions taken January 1, 1923, have not yet been published. It is expected, however, that they will be available for distribution within a few months.

Your Committee wishes to commend the spirit and work of the United States Census Bureau in relation to institution statistics. The officials of the Bureau are in hearty sympathy with our plans for uniform statistics in the institutional field and are always ready to coöperate in any movement for the betterment of such statistics.

Respectfully submitted,

EDITH ABBOTT
KATE H. CLAGHORN
EDITH M. FURBUSH
JOSEPH A. HILL
ROBERT M. WOODBURY
HORATIO M. POLLOCK, *Chairman*
Committee on Institutional Statistics

Report of the Committee on the Proposed Encyclopedia of the Social Sciences

This project grew out of a suggestion made at the meeting of the American Sociological Society in December, 1923. A memorandum adopted at that time stated:

> It is significant that scholars in various branches of social science feel the need of an authoritative summary of results in related fields. . . . The social sciences have outgrown the stage of schools of philosophy. They are now ready to begin organizing many scattering items of knowledge into a cogent body of principles.

In 1924 and 1925, committees have been at work representing the American Economic Association, the American Sociological Society, the American Anthropological Society, the American Statistical Association, the American Association of Social Workers, the American Political Science Association, and the American Historical Association, the last not having yet officially voted participation in the plan.

After a thorough study of the whole situation, the Joint Committee reports to the various constituent organizations its main conclusions as follows:

1. Such an encyclopedia would include all those sciences which are either primarily social in character and content or which have certain social connotations. In the second class the material to be included in the encyclopedia would consist only of those aspects of the subject which are of special importance to the social student. It is the inter-relation of the sciences with the more specific social sciences that it is especially important to emphasize.
2. To the question whether such an encyclopedia is desirable the answer is an unqualified affirmative. It is indeed true that many of the sciences in question are still largely tentative in character, but much can be gained from a full statement of present knowledge and achievement as a basis for progress.
3. Is such an encyclopedia feasible? This again is answered in the affirmative in the belief that there are a sufficient number of editors and contributors of high scientific standing to justify such a project.
4. For whom should such an encyclopedia be intended? Two classes would have to be considered: (1) Primarily for scholars in search of both factual and methodological information; (2) a larger public composed of editors, professional men, and others who need a standard book of reference because of the intimate relation of the social sciences to the problems of today, which are faced in many different vocations and professions.
5. Shall the encyclopedia be primarily a dictionary or a handbook? The suggestion is that it ought to be both in the sense that (a) the alphabetical method would be followed, but (b) that the arrangement would be so flexible as to permit not only brief articles but also longer contributions of original and substantial material. General surveys would be included on the progress of the various sciences and their inter-relations. It is also

suggested that biographies would be published and that brief bibliographies should accompany each major article.

6. What should be the size of the encyclopedia? The Committee has in mind ten volumes of about 800,000 words each, to be published at an approximate cost of $7.50 a volume. The cost has been estimated conservatively at $550,000, this covering $200,000 for remuneration of contributors; editorial outlays of $40,000 a year for six years, or a total of $240,000; and expenses of manufacture and distribution about $110,000. The method of financing remains to be determined. The suggestion is that a large publishing house may undertake this project if there were a minimum sale of 2,000 copies and that supplementary assistance could be sought from individuals or foundations.

   Much remains to be done to develop the details of the project in order to make possible the working out of a financial policy.

The Committee therefore asks the following action by the American Statistical Association: That this report be accepted and approved; that the Committee be continued for the purpose of elaborating the project further; that an appropriation of $250 be made for 1926 to the Joint Committee for necessary expenses, with the understanding that if the project is finally consummated the sum so advanced will be reimbursed out of editorial expenses; and that the Committee be empowered to ascertain how many copies might be subscribed for by the membership of this Association at a reduced rate in order to permit a guaranty to a publisher.

The Executive Committee in charge of the project is composed of E. R. A. Seligman, chairman, representing the American Economic Association; Clark Wissler, the American Anthropological Association; A. A. Goldenweiser, the American Sociological Society; William F. Ogburn, the American Statistical Association; Mary Van Kleeck, the American Association of Social Workers; Carlton J. H. Hayes, the American Historical Association; and John A. Fairlie, the American Political Science Association.

Respectfully submitted,

MARY VAN KLEECK, *Chairman*
R. H. COATS
WILLIAM F. OGBURN
Committee Representing the American Statistical Association

Report of the Statistical Delegation on the Committee of the Social Science Research Council

(The Report of Charles E. Merriam, President of the Social Science Research Council, is printed in full as a separate note in this issue of the JOURNAL on p. 68.)

Committee on Educational and Professional Standards for Statisticians

In May, 1925 the Committee decided that, as a first step, it would be desirable to determine the nature and extent of statistical instruction in American colleges and universities. Accordingly, a schedule was sent to the larger colleges and universities requesting information on the courses given in statistics and allied subjects. More than one hundred institutions responded. A digest of the information thus secured was presented to the Association at its meeting on Tuesday evening, December 29, 1925.

In coöperation with Professor R. E. Chaddock, President of the Association, two meetings were arranged for Tuesday, December 29, 1925 on the subject of instruction in statistics. At the afternoon meeting, held at the Hotel Pennsylvania, New York City, the viewpoint of employers was presented. Papers were read by Messrs. Donald R. Belcher, Carl V. Snyder, Edgar Sydenstricker and Edwin W. Kopf, on the aspects of statistical training which appeal to employers of statisticians. The papers dealt with the service and technical preparation of statisticians in business establishments, in public health, medicine, and biology, and in insurance. These papers were discussed by Messrs. J. E. Pogue, Joseph Barber, Ernest Katz, Leonard P. Ayres, Carroll W. Doten and M. Albert Linton.

In the evening, a meeting was held at which the viewpoints of teachers of statistics were set forth. Papers by Professors H. L. Rietz, A. R. Crathorne, and W. I. King were read, and were discussed by Professors Horace Secrist, B. D. Camp, Harry Jerome, and by others engaged in the teaching of statistics. Professor Glover presented the main findings of the Committee's inquiry into the teaching of statistics by American colleges and universities.

The Committee hopes to arrange for the publication of the papers presented at the two meetings. The proceedings should interest the several groups of statistical workers in business, public health, insurance, and in economic research. Professor Glover is planning an extensive report based upon the Committee's questionnaire. During 1926, the Committee will consider constructively the problem of standards for statisticians, a project which should interest the large number of students now preparing themselves for careers in statistical work.

The Committee:

James W. Glover, *Chairman*
Edmond E. Lincoln
Robert E. Chaddock
Willford I. King, *Advisory member*
W. A. Berridge
E. W. Kopf
Leonard P. Ayres, *Advisory member*

REVIEWS

The French Debt Problem, by Harold G. Moulton and Cleona Lewis. (Publication of the Institute of Economics, Washington, D. C.) New York: Macmillan Company. 1925. 453 pp. and index.

The Financial Crisis in France, by the Honorable George Peel. London: Macmillan & Company, Ltd. 1925. 317 pp. and index.

The bewildering succession of political crises in France and the catastrophic fall in the exchange after the unsuccessful negotiations on the American debt have increased the constantly widening circle of persons who are honestly seeking authoritative guidance in forming an opinion on French finance. In recent weeks two penetrating and candid studies of French finance have appeared, one in England and the other in the United States—naturally enough. It is doubtful if any such objective analyses could have come from French economists. Capable as many of them are, they seem to be too close to the scene to gain the proper perspective.

These two books represent the mature conclusions of thorough students, based inevitably upon much the same statistical data. In their handling of these data they illustrate perfectly one of the outstanding differences between English and American economic approach. Your true British economist is sparing of his statistics, he avoids the use of tables of figures, they are his ultima ratio. The American, on the other hand, is the devotee of figures, he showers them like arrows about his reader's ears, and, not satisfied with that, accumulates reserve arsenals of statistics which he places in an "Appendix" as a warning of what is in store for any reader whose resistance is not demolished by the first onslaught. Apparently the result is that the British economist reaches and influences a wider audience; while the American writer on the same subject is likely to aim at fewer and more specialized readers who, whether they accept or reject his thesis, will find his book a source of statistical material for their own particular requirements. All American economists are statisticians, but not all British economists. It is a fundamental difference of attitude and approach. The American reader of Mr. Peel's *Financial Crisis in France* will find profound economic analysis presented with vigor and literary charm; but he will miss the familiar clatter of statistics. The British reader of *The French Debt Problem* will be impressed with the orderly development of an argument supported by a mass of statistical data, but he will find the book "hard reading," unless he is more or less directly interested in the subject. Each book was written for its own audience, and each will be read to some extent by the other's audience; which will thereby gain not merely knowledge of French finance, but how its cousin's mind works in approaching any financial-economic problem.

It is an incidental coincidence that the impression left by the two books should be absolutely different. One lays down Mr. Peel's *Financial Crisis in France* with the feeling that France can muddle through without things getting much worse unless—; and closes Dr. Moulton's *French Debt Problem* with the

conviction that there is no hope unless—. It will be noticed that neither authority was willing to commit himself to definite forecast, and very properly, for both books demonstrate the number and variety of factors on which the eventual solution hangs, and emphasize the fact that in the last analysis the solution lies not in the hands of economists but of politicians.

The titles of the two books could appropriately have been exchanged. A considerable part of Mr. Peel's book is devoted to an historical analysis of the financial troubles of France, tracing the origins of the present difficulties far back before the war. Candidly and without acrimony, he explains how and why the French Government up to 1917 financed "the most expensive of all wars without requiring the French taxpayer to contribute a single penny," and analyzes the development of the post-war debt with a sincere appreciation of its tragic meaning. His narrative moves forward with the swiftness of a Greek drama, with the climax at the failure of the Reparations Program. He offers no panacea, proposes no solution. He presents the balance sheet, with all its assets and liabilities, luminously, candidly, and sympathetically. His book is not argument, it is exposition, with the vigorous movement of a narrative. And, incidentally, there is humor without levity, and wit without mockery. No one who reads this book will ever again refer to economics as a "dismal science."

Like *Germany's Capacity to Pay, The French Debt Problem*, by Dr. Moulton and Miss Lewis, is argumentative. The authors set out to prove that French Government finances are in a bad way, indeed hopeless, unless certain drastic, not to say violent, remedies are applied—measures which, though financial, would require political action. Having demonstrated this, they present a chapter on French politics and politicians, which leaves the reader thoroughly convinced that no salvation can be looked for from that quarter. It is a discouraging picture, and unfortunately, very persuasive.

Briefly, the argument runs as follows: The current French budget, even with practically no allowance for payments on the inter-allied debts, is not balanced, and further, cannot be balanced because it is in excess of France's capacity to pay. That is, if balanced, it would require taxation in excess of that fraction of the national income which can be attributed to the state without either impairing current production or provoking open rebellion. The authors believe that both England and France are at present taxed virtually to the limit of endurance. Since they find little hope of substantially increasing the income of the French Government, the alternative is to reduce expenditure. The conclusion is reached, after taking into consideration all items of the budget, that the only element susceptible of radical reduction is that of the debt charge, through the compulsory refunding of the internal debt on a two per cent basis, a solution which is scarcely practicable except as the sequel to a revolution. The alternative to the reduction of the debt charge by refunding is reduction of both principal and interest by inflation, which the authors reject.

Having demonstrated that the internal debt is at present an intolerable burden to France, the authors logically devote comparatively little space to the external debt, although they offer a very careful analysis of the French balance

of international payments. Their conclusion is exactly opposite to that reached in the case of Germany. France, they believe, has a balance of payments so favorable as to support a very large transfer of funds, provided the French Government could so adjust its budget as to acquire control of the needed sums.

As was said above, the book is distinctly argumentative, and argument invites rebuttal. The keystone of the whole argument is the estimate of the national income; and the calculated figure of the national income, although derived by close reasoning and based upon excellent authority, is certainly open to dispute. The reviewer is also inclined to believe that in the estimate of the credit side of the "balance of payments" too little weight is given to the fact that under depreciating currency, large merchandise exports represent the liquidation of capital assets at a severe loss. Once the franc is stabilized, it would not be at all surprising if the international balance of payments should turn seriously adverse. The experience of other countries that have stabilized severely depreciated currencies certainly suggests that the immediate sequence to stabilization is a very large increase in the ratio of imports to exports. The reasons for this are involved, but are probably due to the under-consumption of populations afflicted with depreciating currencies. Although the terms "inflation" and "deflation" form a large part of the stock in trade of all economists, it is an unfortunate fact no one really begins to understand either their social or economic consequence. The next generation of economists, living, let us hope, under conditions which will give those terms merely an academic and historical interest, will be able, at their leisure, to unravel the mystery.

In the three months after these books appeared, franc exchange fell by about twenty-five per cent. To some extent this fundamental and rapid change vitiates the conclusions involving the value of the franc and imposes on the reader the task of a good deal of mental recalculation of the statistical data. It does not, however, impair the intrinsic value of either book. In fact, it tends to corroborate Dr. Moulton's thesis that the inescapable alternative to his program of forced refunding was further deprivation. As was stated above, *The French Debt Problem* argues to a conclusion that some will accept and some will reject. Both parties will agree, I think, in their recognition of the fundamental soundness of Dr. Moulton's approach to an analysis of his problem, and those who deny his conclusions will have to dispute the ground step by step. And even they will find it useful to borrow from his arsenal most of the weapons, in the shape of fact and statistical data, which they will require for the contest.

ROBERT B. WARREN

Washington, D. C.

Principles and Methods of Statistics, by Robert E. Chaddock. Boston: Houghton Mifflin Company. 1925. xvi, 471 pp.

It was with misgivings that the American Statistical Association yielded to the insistence of Professor Chaddock that he resign from its secretaryship. His insistence has since been justified by two developments—first, the discovery of a

man who would properly maintain the standards of the secretariat, and second, the use by Professor Chaddock of his new-found freedom to make available to others the results of his many years' experience in the teaching of elementary statistics.

Principles and Methods of Statistics is designed for a one year elementary course in statistics in colleges and universities. It is concerned with statistical methods in general, and not those pertaining to any single field. Its purpose is clearly expressed in the preface, as follows: "To introduce the subject to beginners by the use of simple illustrative materials, to present the elementary principles of statistics in such manner as to appeal to the logical faculties of the reader, and to foster a healthy skepticism toward the results of quantitative investigations." This last purpose, the desire to promote a healthy skepticism, is too often neglected in the current teaching of statistics, although several years ago, one course was described to the reviewer as "seventeen lectures in which we were told what not to do, and one in which we were told what we could do." Professor Chaddock introduces the reader to the pitfalls of statistics in Chapter II, Misuses of Statistical Data, and later sections on limitations on the experimental method, the unreliability of an average, limitations of the moving average, and the like. The frequency of such sections of warning testifies to the author's eagerness to develop the critical faculty in his readers.

Since the main body of the book must necessarily cover the same ground as that of other statistical texts, one is particularly interested in Professor Chaddock's approach. For this reason, Part I is perhaps the most significant section in the book, and one regrets that it is not longer. The three chapters in this part—Chapter I, The Appeal to Facts, Chapter II, Misuses of Statistical Data, and Chapter III, Statistics in the Service of Science—are well worth the inspection of statisticians many years beyond the elementary course. They are not technical but rather philosophical. They present statistics as a method—as the scientific method for use in the social sciences. They are exciting and provocative. Students should at least enter upon their technical trials with eagerness.

Instead of following the order which at first glance might seem more natural, Professor Chaddock discusses the classification and description of mass data in Part II and the gathering and presentation of data in Part III. The author notes that this order has been justified by his experience, as being the most satisfactory method of building on the previous experience of students who already possess some knowledge of sources, and will not recognize the importance of collection and presentation until they have struggled to analyze data.

Part II contains the main discussion of statistical methods, and covers the problems of the frequency distribution, averages, variation, index numbers, probable error, correlation, and time series. It is difficult to believe that one book can possibly cover so much ground adequately, yet the discussion often extends beyond matters of procedure into criticism, evaluation, and uses. There are numerous examples chosen from fields sufficiently wide to demonstrate the full scope of statistical subject matter. Short methods and practical suggestions are often included.

In general, the approach to specific problems is empirical and logical rather than mathematical. Little previous mathematical knowledge is required. Not even acquaintance with the analytical geometric concept of the straight line is assumed. Mathematical formulae do appear in the text but logical rather than mathematical derivations are included, the mathematical derivations often being included in footnotes, where the use of fine print somewhat reduces their ability to terrify. Professor Chaddock has contrived to avoid the difficulties of complicated mathematics with happy success.

In comparing this text with nine others in current use, certain interesting features appear. Professor Chaddock has given more space to the problem of averages than has any of the other authors. Elaborate and relatively thorough treatment is given the arithmetic average, the median, and the mode, and the geometric mean is briefly presented. Although Professor Chaddock does not discuss this point, the reviewer wonders if perhaps the explanation may lie in the fact that Professor Chaddock is avowedly writing a *general* text, and that the importance of the average is chiefly in non-economic fields in which many of the other texts are less interested. The discussion of probability and the probable error is also more extensive than in most other similar texts. On the other hand, the discussion of index numbers occupies space considerably below the average, due chiefly to the fact that there is no critique of current indexes so often found in statistical texts. The chapter on graphs is much shorter than in other texts, although it wisely includes the report of the Joint Committee on Standards of Graphic Presentation.

It should be noted that there is practically no discussion of interpolation or of estimation, nor does the reviewer feel that the sections on sampling are adequate. No attempt has been made to deal with logarithms, partial or multiple correlation, or curve-fitting other than straight lines. Although these last two may well be omitted as too complex, it is difficult to understand why logarithms have no place. Perhaps it is assumed that the students will have such knowledge. If so, it might well be employed to suggest correlation and curve-fitting by using the logarithms of the data, in solving certain problems where linear solutions are not applicable.

There are very few points where the reviewer, in spite of his attempts to do so, can find serious fault with this text. Perhaps the section most eligible for harsh treatment is that on secular trends. From its very nature, a non-mathematical text is in trouble when it reaches this point. Professor Chaddock has limited his discussion to the moving average and the straight line as methods of computing trend. At least one more qualification should be added to those which he notes concerning the moving average, namely, that it always lies inside a curve. For example, the moving average of a semicircle lies inside it, never touching the original curve. Consequently, as used on page 335 to depict the trend of British wholesale prices, 1875–1915, during which period the curve was decidedly bowl-shaped, the moving average is above the proper trend line. It might also be worth while noting that some trends can be removed by changing the units, as, for example, using per capita figures instead of total volume.

The matters of form so important in a textbook have been well cared for.

The book is well printed, durably and attractively bound, and is of convenient size and weight. The charts and tables are excellent examples of what statistical presentation should be. There are many sub-headings, section titles, and an adequate index. Each chapter is concluded with two lists, one of "Readings" and one of "References." These are entirely theoretical in nature, and it might be helpful to include a list of notable applications of the methods described in the chapters.

Professor Chaddock's influence, so strongly impressed on the present generation of statisticians through contacts in the American Statistical Association, will perhaps be felt even more strongly by the next generation, through this excellent textbook.

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Forecasting the Price of Hogs, by Charles F. Sarle. Babson Prize Essay published in the *American Economic Review*, Supplement 2, Vol. XV, No. 3, September, 1925.

This essay, which received the award as the best study of price forecasting during 1924 by any student, graduate or undergraduate, at any American university, demonstrates once more that thorough knowledge of an industry is as important as is statistical technique alone if worth while results are to be obtained. In presenting the significance of the hog enterprise to the American farmer, in tracing the way in which the size of the corn crop as well as the price of hogs affects future production, and in actually forecasting the movement of hog prices, Sarle shows clear insight into the nature of the industry involved and an admirable workmanship in presenting his facts, even though from the point of view of statistical technique the correlation analysis employed is not so satisfactory.

He points out the variations in corn yields, and the factors responsible for those variations; the relation of corn yields to corn prices; the significance of corn prices to the hog producer; and the final effect upon subsequent hog supplies of farmers' responses to changes in corn prices. Except during the war years, all the major changes in the direction of hog prices since 1900 are traced back to antecedent changes in the relation of corn prices to hog prices.

Multiple correlation is then employed to obtain a forecasting equation in which business activity, corn prices, hog prices, and the corn-hog ratio are all given proper weight. Unfortunately, the data were not set up in such a way as to give the most effective results. The four independent factors included in the regression equation (with various lags from three to twelve months) gave a multiple correlation with hog prices of 0.75, while a single one of the independent factors—hog prices of six months previous—gave a correlation of +0.71. In other words, the three other factors which were added to the regression equation reduced the

error in estimating future hog prices from current ones by only 11.7 per cent.[1] Sarle recognizes this fact in his statement that using current hog prices as a factor "has the disadvantage of delaying the change in direction of the predicted price by about six months"; but apparently fails to see that the price given by the regression equation is largely only the current price itself shifted forward six months along the trend line.

It is interesting to inquire why the statistical study is thus less satisfactory than the qualitative analysis upon which it was based. The difficulty seems to lie primarily with the way the corn-hog ratio factor was measured. The four factors represented in the regression equation accounted for 56.7 per cent of the total variation in price, but the corn-hog ratio factor contributed only 2.8 per cent to this total, or about 5 per cent of the total ascribed to the four factors.

The way in which farmers react to the changing profitableness of hog production (of which the corn-hog ratio is a measure) is the fundamental cause of the recurring swing in hog production and is therefore one of the fundamental factors in predicting hog prices. It would seem that the slightness of the forecast was due to the lack of proper weight for this factor. With the monthly data swinging from crest to trough in from 12 to 24 months, the use of a 12-month moving average apparently eliminated much of the significant variation in the "corn-hog ratio" factor.

Fortunately Sarle did not let the statistical results lead him astray. As he says in his article, "Fully as much utility will be obtained from a careful study of the possible causes back of wide departures of the predicted from the actual price in the past (as from the correlation study). If the widest deviations from the predicting formula are studied with great care, common sense judgment can be developed as to special factors at work in the market from time to time."

From his examination of the hog prices themselves, corrected for seasonal and trend, Sarle concludes: "When hog prices deviate more than twenty-five per cent from their secular trend, corrected seasonally, for two months or more at a time, there is a pronounced shoot in the opposite direction and to the opposite side of the prediction line in about eighteen months." This tendency to rebound is due, of course, to the tendency of farmers to reduce breeding at times of excessively low prices. On the basis of this tendency, as well as from his correlation results, he came to the conclusion (in the fall of 1924) that in late March or early April hog prices would reach ten or eleven dollars, and that during August and September, 1925, heavy hogs would sell above $12 at Chicago. In August, 1924, when this forecast was made, heavy hogs sold just below $10. The following

[1] The total gain in accuracy of estimate from adding a group of factors to a previous correlation may be determined from the respective standard errors of estimate obtained from the two correlation coefficients, from the relationships (Yule, *Introduction to the Theory of Statistics*, pp. 247, 248).

$$\sigma^2_{1.2345} = \sigma^2_1(1-R^2_{1.2345})$$

and $$1-R^2_{1.2345} = (1-r^2_{12})(1-r^2_{13.2})(1-r^2_{14.23})(1-r^2_{15.234})$$

for $$\sigma^2_{1.2} = \sigma^2_1(1-r^2_{12})$$

hence $$\frac{\sigma^2_{1.2345}}{\sigma^2_{1.2}} = \frac{1-R^2_{1.2345}}{1-r^2_{12}}$$

or for the present case, $$\frac{\sigma^2_{1.2345}}{\sigma^2_{1.2}} = \frac{1-(.75)^2}{1-(.71)^2} = .823$$

and $$1.00-.823 = .117$$

March they averaged $13.70; in April, $12.60; and in August and September of 1925 about $13. Thus the market reacted about six months earlier than was forecasted. Adding knowledge of his subject, common sense, and good judgment to statistical technique, Sarle has, however, scored high in the difficult business of actual price forecasting.

One note of caution must be sounded with regard to drawing conclusions from studies of this type. The only immutable law of our economic order is the law of change; the influences which determine a price in the past will not necessarily continue to affect it in just the same way in the future. The hog market in recent years is itself a case in point. The pig survey, initiated by the Department of Agriculture in 1922, has so changed the response of the market that what was formerly substantially a simple commodity market now has some of the characteristics of a speculative futures market. Until sufficient data are available to analyze the reactions of the market to this new factor, the knowledge gained from studies of the earlier record must be used with care. Other less striking shifts—in dietary habits, in economic inter-relations, in foreign demands—are constantly going forward. Recognition that change *is* inevitable should prevent our making the assumption, express or implied, that events in the future will follow exactly the patterns of the past.

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An Introduction to Mathematical Probability, by Julian Lowell Coolidge. New York: Oxford University Press. 1925. xii, 216 pp.

When one considers the number of books on statistical method that have appeared in recent years, the paucity of modern texts in English on the related—some would say the parent—subject of mathematical probability is somewhat remarkable. Of the two that have been published Fisher's admirable book develops mainly the statistical aspects of the subject, while the brilliant but perhaps rather rash treatise of Keynes is devoted to an examination of the philosophical basis of probability and treats the mathematical side of it only cursorily and in the main destructively. There has been need, therefore, for a text, such as the present, to serve as an introduction to the whole subject of mathematical probability.

After an introductory chapter in which he discusses various proposed definitions of probability and adopts that which defines it as the limit to a statistical ratio, Professor Coolidge treats permutations and combinations, total and compound probabilities, expectation, risk, Bernoulli's theorem and the probability integral, mean value and dispersion, geometrical probability, Bayes' theorem, errors of observation including errors in many variables and indirect observations, closing with chapters on statistical mechanics and insurance. His style is picturesque and usually clear, and his explicit statement of fundamental assumptions and emphasis on the approximate character of many of the

formulae is most praiseworthy. I am inclined, however, to believe that he is rather sanguine in his estimate of the mathematical fluency of students of his book. I do not mean to suggest that he should have written a non-mathematical text on mathematical probability, but his Introduction would have been more useful to many readers if it had reflected more of the viewpoint of the applied mathematician.

In the discussion on page 194 of Makeham's and Gompertz's formulae there seems to be a certain confusion of thought. The real empirical test of these formulae is whether when fitted to a particular life table they represent *that* table accurately. Unless the Gompertz formula in the figures quoted from Bertrand was fitted to the British Companies table the comparison is quite meaningless. In any event Deparcieux, Duvillard and Northampton have nothing to do with the case. Moreover there is no reason to consider any given life table as "best" in an absolute sense. A life table represents the conditions of mortality of a particular population, limited in space and time. It is true that the method of construction of the early life tables did not allow for the effect of a changing population, but in all probability if we had the data for applying the more refined modern methods to eighteenth century populations the resulting life tables would still differ markedly from modern experiences simply because the conditions of mortality were different.

There are a number of printer's errors in the book, but these are fortunately easy to detect for the most part.

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Theory of Accounts, Volume I, by D. R. Scott. New York: Henry Holt & Co. 1925. 284 pp.

The author has designed this volume as a textbook for an introductory course in accounting running through one semester. It is divided into four main sections, as follows:

Part I—General Introduction
Part II—The System of Accounts
Part III—Proprietorship
Part IV—Valuation

The statement method of approach is adopted. That is, before consideration is given to the underlying sources of accounting data and the mechanics of bookkeeping, the purposes of accounting—particularly as reflected in the construction and interpretation of balance sheet and income sheet—are discussed at some length (about ninety pages). Throughout, the author lays emphasis upon general principles as opposed to technical details. A feature of the book is the extent to which economic reasoning is employed, particularly in the chapters dealing with valuation problems. The material is interestingly presented, and the style is unusually clear.

In the opinion of the reviewer this book will prove of value as a text primarily

in the case of short courses in accounting given in colleges of literature, science, and arts, where the aim is to furnish a general knowledge of the uses and processes of accounting to students of economics and allied subjects who have no intention of specializing in accounting or in other technical fields of business administration. Not many teachers of accounting who are interested in offering a thorough course in principles as a basis for advanced work, it is believed, will find this material satisfactory. The book is hardly more than an outline; many matters of importance receive bare mention. In particular, the treatment of accounting technique is inadequate for the beginning student who expects to specialize in accounting. The author's position that the instructor can and should supply a great deal of explanatory and supplementary material in the classroom is well taken; but it does not follow that—given the best of instruction—the student is not greatly advantaged who is supplied with text material which includes systematic and full discussion and illustration of all difficult matters.

As a statement of accounting theory the book is fairly orthodox. At the same time Professor Scott refuses to allow himself to be bound by mere convention and rule of thumb. In general he insists upon the importance of market values as a basis for accounting valuations, a position very much to be commended. It is also refreshing to note that his treatment of the subject involves a clear-cut recognition of the character of such contra valuation items as allowances for depreciation, discounts on securities, and special losses.

In view of the excellent general tone of the book it is somewhat disappointing to find (1) numerous unsupported sweeping statements with respect to debatable points, (2) the acceptance of several conventional yet invalid doctrines, (3) a considerable amount of incomplete and rather questionable reasoning. The statements that "dividends on stock do not accrue . . . as an income for the holder of the stock" (p. 166); that "bonds . . . bought . . . as a long-term investment . . . should be kept on the books at a value which reflects their purchase rate"; and that "it is not desirable to attempt a general discounting of accounts receivable" (p. 255), are examples of the first. It may well be urged that it would not be unreasonable accounting for a holder of a high-grade preferred stock, with a long, unbroken dividend record, to give definite recognition to dividend expectations at the close of the fiscal period; that the failure to take account of a long-continued and serious fall in the value of long-term railroad bonds held by an insurance company, for example, is objectionable at least on the ground that the rate of return realized on book values is quite out of line with market investment rates; and that reasons for the valuation of book accounts at maturity figures as opposed to current discounted values should be offered.

Examples of highly questionable doctrines accepted by Professor Scott are the view that sales discount is homogeneous with operating expenses (p. 55), that expenses are direct deductions from proprietorship (p. 74), that stock which has been issued and reacquired is quite distinct for accounting purposes from unissued stock (p. 157), that interest is in operating expense (p. 100), and that estimated interest during construction is a valid property charge (p. 266).

A few examples will also be noted of rather unsatisfactory explanations and reasoning. First, the author indicates (p. 28) that the income sheet is founded

on an intrinsic equation as is the balance sheet, a proposition of very dubious validity. Further, the form of income-sheet equation given here is unreasonable. The income statement naturally reads, gross income minus expenses equals net income. Second, it is not quite correct to hold (see pp. 29–30) that the differences between two successive balance sheets may be accounted for by an examination of the income sheet for the intervening period. Borrowings, retirements of liabilities, changes in proprietary investment, distribution of surplus, and several other types of transactions affect the balance sheet but are not reflected in the ordinary income sheet. Third, in view of the emphasis upon corporate proprietorship as distinct from corporate liabilities it is rather surprising to find the author acquiescing (p. 35) in the proposition that capital stock, bonds, and notes are "long-term obligations of the firm." Incidentally it may be noted that the use of the word "firm" to designate the corporate entity is rather objectionable. Fourth, the distinction between fixed and current assets is not clearly drawn. In this connection it may also be noted that "the use of capital funds to purchase current assets" (p. 37) is not unreasonable in the case of a business requiring a large proportion of current resources. Fifth, the view that "asset accounts . . . are positive accounts" and that "proprietorship and debt accounts are negative," which the author makes the core of his explanation of debit and credit usage, is quite unfounded in fact and it is hard to see how it can be of any assistance to the student.

One or two matters of more general interest should be commented upon. As already noted, Professor Scott emphasizes roundly the importance of market prices in inventory valuations. On the other hand he takes the position (p. 195 and elsewhere) that the income figure should in nowise be affected by the use of market price as a basis of valuation. He even goes so far as to argue that appreciation realized by sale is not, in the case of a continuous business, actual income for accounting purposes, on the ground that the funds representing the appreciation will not be available for distribution but will be required to replace stock. In the opinion of the reviewer this is unsound. In the first place it may be urged that to the extent that market price has no effect upon income it has no significance as a balance sheet figure. The balance sheet purports to represent valid asset values, and such asset values can only be created through investment or earnings (ignoring the possibilities of gifts, thefts, etc.). An earning may, of course, be extraordinary in character, but it is none the less an item of income. Second, the implication that book income must be equivalent to distributable income is certainly unwarranted. The fact that an advance in prices requires the utilization of additional funds to maintain stock in nowise justifies the conclusion that such additional funds cannot be counted as legitimate book income. It is true that ordinary accounting income does not precisely measure increase in purchasing power, on account of the movement in the value of the dollar. It is also true, however, that the particular change in price affecting the inventory of the specific business is not likely to correspond exactly to the change in the value of money, and that some adjustment of actual purchasing power is almost certain to be involved in any such revaluation. And apparently the author is not suggesting a sweeping revision of accounting methods designed to set up as book

income for the business enterprise, period by period, a precise expression of the increase in purchasing power accruing to the enterprise. Indeed, any such suggestion would be extremely academic as long as no consideration is given to the change in the value of money in business contracts, in the definition of taxable income, and in other vital connections.

Any legitimate asset value may become a reservoir for income. And the profits represented by increase in value of inventory are no more likely to be wiped out than the profits resulting from sales which have been used to purchase additional units of stock.

In his discussion of depreciation Professor Scott raises in a new form the old proposition that a straight-line depreciation policy will result in the accumulation of excessive charges and a secret reserve in the case of a property consisting of a considerable number of units which, when settled operating conditions have been established, have an average service life remaining of fifty per cent. The pros and cons of this difficult question cannot be discussed here. The reviewer would like to state, however, that in his opinion the straight-line plan will not result in any serious understatement of property and overstatement of operating charges, assuming that the effective service lives of the various component parts of the property can be forecast with reasonable accuracy. The truth of the matter is that a *smaller capital investment* is required to produce a given volume of service or other product in the case of a "seasoned" property than in the case of an entirely new plant. The new plant is the more valuable; represents the larger investment, primarily because the requirements for repairs and replacements, at least for a period of years, are less than in the case of the older property. It must also be remembered that in many cases in practice the new plant will actually produce more product, or a higher grade of product (particularly in the case of services), than the old. In other words, there is an accrued depreciation in the case of the seasoned plant, and there is no reason why the amount as calculated by the conventional straight-line plan should be excessive. This conclusion, of course, does not deny the fact that funds may be accumulated through the depreciation charges in excess of replacement requirements.

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An Introduction to the Methods of Economic Statistics, by William Leonard Crum and Alson Currie Patton. Chicago and New York: A. W. Shaw Company. 1925. xii, 493 pp.

This volume is another addition to our rapidly growing list of textbooks on statistical methods. As the title implies, however, the authors have had primarily in mind the needs of those students who are interested in the application of such methods to economic problems and this fact has controlled the selection and arrangement of the topics covered and is responsible for the almost exclusive use of economic statistics for illustrative material. The book is designed for students without extensive mathematical training.

The text proper is divided into three parts of almost equal length: Part I, dealing with the collection, compilation, and charting of statistical data; Part II, covering general methods for the analysis of statistical series; and Part III, devoting particular attention to the analysis of time series. The arrangement is well suited to the needs of economic students.

In Part I the distinction between categorical, time, and frequency series of statistics is properly stressed. The theory of statistics deals principally with the frequency series, but the economic investigator is more frequently concerned with the former types. The authors have sharply differentiated the three types in their treatment of the problems of charting, which has been conducive to clarity in exposition. Their discussion of tabulation might have been improved if they had observed more markedly a similar differentiation in that connection. One detail in the treatment of the charting of frequency series is open to criticism, namely, the plotting of the points for a cumulative frequency series at the mid-points of the class intervals instead of at the class limits. On the whole, however, this part of the book contains a satisfactory treatment of the subject matter with which it deals. Methods are clearly explained and there is almost prodigality in the use of illustrative material in the form of both tables and charts.

Part II, covering general analytical methods, deals with averages, dispersion, skewness, the normal law of error, and correlation. This section, judged according to its utility as a text for students beginning the study of statistics, is the least satisfactory part of the book. Too often the treatment is sketchy and inadequate, providing merely an outline whose value to the student could be much improved by amplification. Of course, considerations of space enter in here, but some sections could probably better be omitted entirely rather than be subjected to such brief discussion as to serve no useful purpose. For example, over a page is devoted to the formulas for exponential and inverse-exponential averages and to the technique of working out such averages, but no explanation is given as to the reasons for the employment of these formulas. In fact, in the whole discussion of averages, dispersion, skewness, and the normal law of error, emphasis has been placed upon the presentation of formulas and upon the technique of statistics such as the employment of short-cut methods. Sometimes the explanation of these methods is likewise short cut so that the student needs to give particularly close study to these chapters. The usefulness of this portion of the text would be greatly enhanced, if, in addition to the outline of statistical technique more attention had been given to an analysis of the logic of statistical methods. The student needs to know not only how to employ various statistical devices, but also why they are employed, what are the limitations upon their uses, and what interpretation is to be placed upon the results obtained. These requirements are not adequately met in Part II. In the chapter on the normal curve of error, for example, attention is directed primarily to the technique of fitting the normal curve to data. There is, however, no discussion of the theory of probability and, while several formulas for probable error are given, the treatment is not such as to make clear their significance. In the chapters on correlation a good explanation is given of the meaning of straight line correlation and of the formula for measuring this; but it is rather surprising that no caution

is sounded as to the significance of probable error in the results obtained or that no comments are made upon the interpretation to be placed upon the coefficient calculated.

But if Part II is to be criticized for the inadequacy of its treatment of the material included, Part III, on the other hand, may be praised for the very qualities in which the former is deficient. One reads through this section with the feeling of being served with much more than the bare bones of formulas. Statistical methods are discussed in the light of the problems presented. The whys and wherefores, as well as the hows, are considered. This part of the book includes a treatment of index numbers and the analysis of a time series for the determination of secular trend, seasonal variation, and cyclical fluctuations. The methods studied are largely those which have become familiar through the publications of the Harvard Economic Service. In the analysis of secular trend detailed explanations are given concerning the fitting of a straight line to the data and to the determination of the compound interest curve. The methods of fitting a parabola or more complex curves are not taken up. The treatment of the problem of seasonality is practically confined to a discussion of the link-relative method developed by Professor Persons. The chapters on index numbers are restricted to a study of the principles and problems involved in the construction of index numbers of prices. No attempt is made to analyze any of such index numbers actually in use and no detailed consideration is given to other forms of index numbers, except that the work of Professors Day, King, and Stewart on index numbers of physical volume of production is mentioned. While the scope of the discussion in this part of the book has thus been limited, this very fact has made possible a more illuminating exposition of the subject matter to which the authors confined themselves. The explanation of methods is clear and at the same time the underlying logic of these methods and the limitations on their use are set forth with painstaking care.

It is worthy of note that approximately equal space has been devoted to each of the three parts which have been considered. Perhaps this is indicative of the authors' opinions as to their relative importance in a text on economic statistics. In a general text on statistical method the subject matter of Part II would bulk largest, but in a text intended primarily for economic students it is proper that special attention should be given to the problems of compiling and presenting statistical material and to the study of time series. Most of the statistical work in economic investigations is of the latter types. Moreover, the developments in the making of index numbers, and in the analysis of time series, especially in connection with the business cycle, have constituted some of the most notable recent advances in the methods of economic research. Hence these questions are naturally emphasized in a text on economic statistics.

In the appendix are contained numerous tables that may be used as a basis of exercises for students or for illustrative purposes. There is also a valuable statement of certain empirical rules for guidance in laboratory procedure and exercises. A four place table of logarithms as well as a bibliography is included.

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Wages and the Family, by Paul H. Douglas. Chicago: University of Chicago Press. 1925. xiv, 290 pp.

In his *Wages and the Family*, Paul Douglas has written an important and useful book. More directly and positively than any other contemporary economist, he has faced the problem of the "living" wage, which he sees—as too many other students of the problem have not—as a problem involving the ability of industry to pay as well as the physical necessities of the recipients of wage payments.

Douglas attacks two time-honored and easy generalizations: first, that the "typical" family of the wage-earning class is one of five—father, mother and three children under 14 years of age; and, second, that while the general level of wage payments is insufficient to afford an adequate living to such a family, it can at any time and as a matter of course be raised to a point where it will do so.

On the first point, statistical data are conspicuously lacking. Professor Douglas presents such reliable evidence as is available, including some figures for working class families from the British census of 1921, the results of investigations in limited areas in France and Belgium, and the findings in three distinct investigations in the United States, one by the United States Bureau of Labor Statistics, covering over 11,000 families, one by the United States Coal Commission, covering 83,877 householding miners, and a third by the Illinois Health Insurance Commission in 1918, which included over 3,000 working class families in Chicago. All of the studies indicated clearly that for the groups included the family of five was not typical, and that in the majority of the cases the number of children in the family was less than three. These data are sufficiently limited in scope to make too dogmatic generalizations of questionable value, but they do serve to throw serious doubt on the previous assumptions as to the size of the workingman's family.

As to the ability of industry to pay, Professor Douglas applies the living wage principle to the gainfully employed population of the United States as indicated in the 1920 census. Allowing each male worker over 20 years of age an income of $1700, which was assumed to be the cost of providing a family of five with the necessities of life in 1920, and such an income to women, and minors of both sexes as would allow individual maintenance, he finds the total wage bill would have been close to fifty-eight and one-half billion dollars. Estimating the total national income in 1920 to have been seventy-one billion dollars, only about twelve and a half billions are left for differential wage payments, interest, profits and rent—a sum that would seem to be inadequate. The conclusion, therefore, is reached that industry could not have borne the burden, in 1920, of what is commonly understood to be a "living" wage.

As a way of escape from this impasse, Professor Douglas suggests the adoption of a system of wage payments based on cost of living for the individual worker with additional allowances for dependents. Readers of the JOURNAL, largely on account of Professor Douglas' contributions to its pages, are familiar with outlines of the plan. Briefly, it involves the payment of the basic wage and the contribution of additional sums by the employer to a central fund from which allowances are to be paid. The amount of the contribution will depend on two

things: the degree of the risk, that is, the probable number of dependents per hundred or per thousand of wage earners, and the number of wage earners on the payroll. This central fund or pool could be handled either regionally or by industries. Each employee would be entitled to receive from it a definite sum for each dependent. Thus, a living wage would be assured, no matter how large the family might be, and this would not be an impossible burden on industry, since there would be no payments for non-existent dependents, as would be entailed by the family-of-five basis. Such a system is in operation, according to Professor Douglas, to a greater or less degree in most of the countries of Continental Europe, and there is persistent agitation for its adoption in England and Australia.

The proposal that such a system should be adopted in the United States is not made by Professor Douglas without recognition of many difficulties, some of which he concedes to be insuperable, at least for the present. It could not be made to work, for one thing, on less than an industry-wide basis. No state has jurisdiction over a whole industry. If it did, the courts would probably find a law establishing such a system unconstitutional, and it is certain that Congress could not do it for industry as a whole, unless it were for the railroads, and, possibly later on, the coal mines. Single men would not be quiescent under a reduction in their wages, while family allowances were being paid to householders. If the plan were to be inaugurated voluntarily, therefore, it would have to start in an industry where the wage level is already on a cost of living basis for a person without dependents. Professor Douglas suggests that this is now the situation with respect to common labor. In a chapter devoted to objections to the proposed system, other difficulties are dealt with.

One must be impressed, in reading this book, with the painstaking effort with which the author has sought out every difficulty and endeavored to meet it. Thus, many apparent obstacles are thrust aside. It seems to the reviewer that some remain unshaken, however, and that these are obstacles of the first importance.

First, the tendency of a system of family allowances, unless it is made compulsory on industry as a whole, must be to put a premium on the employment of the worker without dependents. As long as the system is voluntary, self-interest will lead inevitably in that direction. The formation of an association of employers, with a pool from which the allowances are paid, will modify but will not destroy this tendency. There will be a natural trend toward the elimination of bad risks, i. e., the exclusion from the pool of those employers whose workers have many dependents. According to Professor Douglas, this tendency is to be observed in France, where the plan has received wider acceptance than elsewhere.

Second, under such a plan, the basis of wage payments is not productivity, but necessity. The single man who by reason of skill and the bargaining power of his group has established for himself a wage fairly proportional to his contribution to industry will either suffer a reduction in wages to the point suggested by his "needs," or he will fail to advance as his productivity increases. On the other hand, the family of the married man will receive an income sufficient for

support entirely on the basis of need and regardless of the wage earner's contributions to the processes of industry.

It should be said here that Professor Douglas is clear throughout as favoring the establishment of the living wage as a minimum, and he suggests that the basic wage can be increased above this minimum by collective bargaining. But it is doubtful whether such an outcome could be anticipated with assurance. The plan itself is a divisive element, setting off the workers with no dependents from those having dependents. It seems almost certain that those workers with large families will be weaned away from the labor movement, thus weakening the bargaining power of the remainder. There is also the possibility, which Professor Douglas recognizes, of the pool being used by the employers for disciplinary purposes, as has been done in Belgium. This can be avoided only by compulsion, or through joint control of the funds by employers and unions. But, as far as this country is concerned, the former is impossible, and the latter is unlikely, in view of the probable disrupting effect on unionism of the plan itself. Of course, there is no assurance that the workers are now in receipt of a wage that is justified by their efficiency. But at any rate, the door is open for a combined thrust in that direction, and the obvious need of increasing efficiency if wages are to continue to rise is beginning to be recognized by some of the trade unions. This most encouraging move would have less reason for being if the system of family allowances were put into effect.

Third, the adoption of such a system based on physical need and not on earning power would be likely to destroy labor's independence and self-respect. Instead of a body, as now, actually or *potentially* independent and self-reliant, we should have transformed the workers into a mass of pensioners—recipients, not of earned income, but of something suggestive of *largesse*. The French employers objected to a bill making such payments compulsory, on the ground that "it would mean replacing the employer's bounty by an acquired right." Security of income, at such a price, is too dearly bought.

To the present writer it seems that Professor Douglas has underestimated the possibilities of increased production, and that in his healthy rejection of laissez-faire he has gone to an opposite extreme that is possibly equally objectionable.

JOHN A. FITCH

The Austrian Crown, Its Depreciation and Stabilization, by J. Van Walre' de Bordes, LL. D., with a foreword by Sir Henry Strakosch, K. B. E. London: P. S. King & Sons, Ltd. 1924. 248 pp.

Ever since the war, the writer of this review has been dreaming of a series of studies on the monetary lessons taught by the war. The Napoleonic wars gave rise to the "bullion report," still a classic. The lessons of the World War should be embodied in far more important and more numerous studies. The material already available is wonderfully rich and diversified and most of the war-episode is now over, so far as the various monetary phenomena are concerned. Probably there is not a country in the whole world which has not had some monetary experience the statistical records of which would throw some light on the theory of

money. Gold, paper, and credit inflation, followed by deflation, and stabilization which succeeded each other in rapid succession during the last decade, are awaiting the chronicler and analyst. They afford an unprecedented opportunity for gathering and studying specimens of monetary experience in endless varieties.

There is Czeckoslovakia, which made the unique experiment of using a former currency (Austrian), calling it in and reissuing only half. This experiment has not yet been adequately reported. There is Sweden, which could scarcely believe its eyes when its paper money was worth more than the gold which was supposed to give it value. There is the Ukraine, where many kinds of paper money circulate simultaneously, and where the one having the highest value is outlaw money, having been issued by a government long since defunct. Doubtless other unique specimens of monetary experience can be found.

Just as the present reviewer was indulging in these day dreams, *The Austrian Crown* came to his desk, a startling and complete realization of those dreams so far as one country, Austria, is concerned. The author's presentation is forceful and convincing. Only an extremely carping critic indeed could find much fault with this monograph.

As a member of the secretariat of the League of Nations, the author has had excellent opportunities to study the problem, both at Geneva and Vienna.

The work consists of three parts. Chapters 1 and 2, which are introductory and historical, form the first part; chapters 3 to 6, which are statistical, form the second; while chapters 7 and 8 are "theoretical" and give the chief conclusions.

A large part of the author's argument centers around the "equation of exchange," $MV+M'\ V'=PT$. As this equation of itself does not affirm or deny the quantity theory or any other causal relationship between the quantities connected by this equation, the author undertakes to discover, from the historical and statistical facts at his disposal, to what extent money (M) or deposits (M') controls the price level (P), that is to what extent the quantity theory is justified; to what extent, reversely, the price level controls the money, and to what extent other factors have to be appealed to. He finds examples of all three. Consequently, his findings include a strong but qualified confirmation of the quantity theory—with much emphasis on its inadequacy to explain all of the facts during the turmoil and transition periods under review.

One interesting episode followed the stabilization of the crown. With the return of confidence, and the greater use of money, the price level remained steady although the circulation increased many fold.

Special attention is also given to the connection between the price level and the rates of foreign exchange, a connection which appears to be different at different stages of depreciation.

The author discusses the purchasing power parity theory and other theories of foreign exchange but less conclusively than the quantity theory.

The book abounds in statistical tables and charts, the latter showing clearly to the eye most of the conclusions arrived at. There is a chart showing the close relation between the price level and the note circulation between 1914 and 1918, another for the year 1919 in which very little such connection appears, and other

charts for succeeding years. That for 1922 shows a close relation between foreign exchange and the price level.

Nor are the psychological factors overlooked. During the period of depreciation the author shows that sometimes the distrust of the crown was the chief reason for its fall. The effect of such a fall on the velocity of circulation, the volume of trade, and the use of money in trade, the attempts to use foreign currencies, the flight from the crown, etc., are all set forth with skill and with every evidence of scientific open-mindedness rather than devotion to any preconceived theory.

In closing, the reviewer would like to express the hope that some day a larger part of his whole dream may come true—that other writers from other countries may follow the good example of Bordes and interpret with equal open-mindedness the peculiar monetary phenomena which other countries afford.

IRVING FISHER

Mathematics of Accounting, by A. B. Curtis and J. H. Cooper. New York: Prentice-Hall. 1925. 397 pp.

The outstanding characteristics of this book are the absence of highly developed and involved mathematical formulae and the presentation of methods and short cuts for ordinary every-day problems. Much of the material is plain arithmetic with devices for getting results quickly. This last statement is not made in the spirit of adverse criticism, but rather to indicate the kind of usefulness the book serves.

Very little in the way of general exposition is to be found in the thirty-two chapters. Instead, the method of presentation consists of a number of specific problems and solutions followed by other problems of a similar nature, without solutions. In brief, the book is almost exclusively a compilation of concrete mathematical problems and not a comprehensive treatise on fundamentals.

To give a better idea of the contents, the topics taken up include: arithmetical short cuts, figuring merchandise turnover, interest and discount calculations, logarithms, valuation of good-will, foreign exchange transactions, annuities, bond discount and premium, depreciation, and building and loan associations. Tables for logarithms, present worth, compound interest, and annuities are to be found in the appendix.

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